Volatility Surface and Term Structure-based Modeling and Analysis: High-profit Options Trading Strategies

Shifei Zhou, Hao Wang, Kin Keung Lai, Jerome Yen

Content

Preface

The option trading became popular from the date of Jun 26th, 1973 when the Chicago Board Options Exchange (CBOE) standardized and integrated option contract transaction. The existence of options expands the range of investment choices and explores investment channels for investors. Generally, options can provide investors with good possibility to obtain a higher income. There are many branches about the research on options. The risk hedge functionality of options is also welcomed by investors. The proper understanding and manipulating the Greek risk indicators of options can help investors to measure and manage risk.

This book proposes different financial models based on option prices to predict underlying asset price and designs the risk hedging strategies. The authors review the literature and improve the traditional volatility models. Theoretical innovation is made to these volatility models and makes these models suitable for real market. Besides, risk management and hedging strategies are designed and introduced based on different criterions. These strategies can provide practical guide for real option trading based on the prediction results from theoretical models.

The half of chapters of this book focuses on volatility models and the applications of these models to perform market forecasting. Another half of the book is more oriented towards the risk management and option trading strategies design. The details organization of this book is as follows.

In Chapter 2, the main idea is to use an implied volatilities term structure-based Heston model to forecast underlying asset price. The parameters of Heston model are estimated by least squares method. The term structure is calculated and applied to the Heston model as the long-run mean level. Finally, we simulate price distribution of the underlying assets on the basis of Heston and CEV models.

In Chapter 3, motivated by disadvantages of traditional Heston model, we propose an adaptive correlation coefficient scheme to estimate the Heston parameters. To precisely estimate this correlation, Heston model is trained by least squares method on historical data every day when the underlying price is simulated every day based on the implied volatility term structure.

In Chapter 4, we use basic theories of option pricing and the method of simulation to clarify the significant role that options can play in risk management. The characteristics and hedging effects of options are analyzed by combining options with underlying asset. This can not only widen the method to control the risk but also promote the development of option.

In Chapter 5, we propose to manage risk of investing on equity by applying VaR and CVaR as standard criterions. Besides, a model based on these criterions is built for the design of optimal strategies. Through empirical tests, we find that a good profit can be made when the prediction of the trend of underlying asset price is high efficient and precise. The loss is also effectively controlled when the prediction is bad and inaccurate.

Chapter 6 studies the traditional local volatility model and proposes a novel local volatility model with mean-reversion process. The larger local volatility departs from its mean level, the greater rate local volatility will be reverted with. Then, a Bi-cubic B-spline surface fitting scheme is used to recover local volatility surface. Monte Carlo simulation is adopted to estimate underlying asset price trend. Finally, empirical tests show our mean-reversion local volatility model has a good prediction power than traditional local volatility model.

In Chapter 7, we propose a regression-based dynamic correlation Heston model. The correlation is estimated by three different regression models between the volatility and underlying asset price. They are respectively simple regression model, polynomial regression model, and auto-regression model. The prediction performance is compared among these models in the empirical tests.

In Chapter 8, for the risk management of option, buying or selling the underlying assets is useful to hedge the potential risk in a certain degree. We propose a self-risk management method to control risk. The combination between different types of options is designed as self-risk management. The underlying asset price is predicted and used to back-test our self-risk management method.

In Chapter 9, we propose a novel call-put spread-based model for underlying asset price forecasting. The curves of implied volatility of call and put options are calculated separately. The distance between call and put implied volatility curve contains important market information. We use this distance to predict the underlying asset level and obtain a good result.

Shifei ZHOU
Department of Management Sciences
City University of Hong Kong
Kowloon, Hong Kong
Email: shifeizhou@gmail.com

Hao WANG
Department of Management Sciences
City University of Hong Kong
Kowloon, Hong Kong
Email: haowang_2006@163.com

Kin Keung LAI
Department of Management Sciences
City University of Hong Kong
Kowloon, Hong Kong
Email: mskklai@cityu.edu.hk

Jerome YEN
Department of Finance
Hong Kong University of Science and Technology
Clear Water Bay, Hong Kong
Email: risksolution@gmail.com

December 28, 2012

List of Figures

List of Tables

Chapter 1 Introduction

Stock option trading is a new way of stock trading developed in the 1970s, commonly used in the United States in the early 1990s. The stock option is a right that the company gives the operators this right to buy or sell a certain stock in accordance with a fixed price within a certain period of time. The operators are given neither a cash reward not the stock itself but a right. The operators can buy company stocks under some preferential terms.

The option is a right, which allows buying or selling a certain number of underlying asset shares at a specific time in the future at a specified price. The execution of an option in options trading is a process that the buyer decides whether to buy or sell the underlying asset at the price that the seller and the buyer have settled. And the seller can only passively accept the compliance obligations. Once the buyer asks to execute the option contract, the seller must fulfill obligations and settle the position specified in option contract. Hence, the rights and obligations between the seller and the buyer are not equal. The underlying assets of options include commodities, stocks, stock index futures contracts, bonds, and foreign exchange.

The underlying asset of stock index options is the spot index. Take the current popular European option for examples, buyers and sellers will directly settle the option contract by cash for stock index options when these options expire. For real option trading, the buyer of stock index option will only execute the buy right when the option generates the floating profit (earnings greater than the transaction fees) and give up to execute the right when loss (including the case of earnings is less than transaction fee). Hence, there is less risk for the option buyer.

For stock index options sellers, they will face the risk of loss only when they have to passively execute option contract. However, as long as the premium that sellers collect from buyers can cover the losses, they can hedge the risk of losses. Based on the volatility calculated from the stock index and stock index futures contracts, the option sellers will be able to control the risk of losses within the 95% confidence interval by collecting conservative premium rate of 10% -15 %.

Compared with commodity options, the execution of stock index options and stock index futures are two independent delivery processes. The underlying asset of stock index futures contracts is the spot price index. The stock index is usually composed of a basket of stocks in accordance with a certain proportion. Commonly, the cash mode is used for the settlement of stock index. That is the gains and losses of stock index investors are settled by cash at the maturity date of futures contracts.

The execution of stock index options is similar to stock index futures, which can be divided into two modes, U.S. mode and European mode. Since the underlying asset of stock index options is also the spot stock index, the stock index options will be converted to stock index futures at the execution date (U.S. mode) and exercise with futures, such as the small CME S&P 500 index futures and options. Another popular European option is directly executed with cash at maturity date, such as the CBOE S&P 500 Index options. However, for European options, the market price usually contains intrinsic value and time value. The intrinsic value is the expected value between underlying asset price and strike price at maturity. Hence, in this book, we make use of market option price and do not discuss the intrinsic and time value of option price in details. Option price is also referred as market option price in default.

The options trading based on futures can provide a hedging function for the futures trader. To format a multi-level portfolio, especially the low-risk high-yield portfolio in the futures market, needs to take advantage of the options. To reduce the trading risk of futures investors, to expand the scope of the market participants, and to improve market stability and liquidity also require options. Besides, options can provide hedging tools of risk for contractual agriculture to stable and protect farmers' revenue. U.S. government encouraged farmers to successfully combine the government subsidy with options market so that to transmit the huge risk of agricultural market to the futures market. This policy not only reduces government fiscal expenditure but also stables agricultural production and effectively protects farmers' benefit.

In addition to hedging functionality, the stock index options also contain certain market information from participants. During options trading process, buyers and sellers provide bid and ask price for options. These prices contain views of buyers and sellers about what the underlying asset price will be at the maturity date. Black and Scholes (1973) proposed that these views could be presented by implied volatility. The implied volatility calculated from option price implies market information. If the implied volatility of put option is large, this means that the market participators feel panic about future market. Conversely, if the implied volatility of call option is large, this tells that the market will go up in the near future. Therefore, the study of option implied volatility is significant and beneficial.

(1) Implied volatility

Implied volatility of an option contract is the volatility of the price of underlying asset which is implied by the price of the option. Implied volatility varies with different strikes and time to maturities. For a given time to maturity, the implied volatility varies with strikes; for a given strike, the implied volatility varies with different maturities. To price the European options, Black and Scholes (1973) proposed that the price of underlying asserts satisfies the following process.

$$\frac{dS}{S} = \mu dt + \sigma dW$$

where μ is the mean value of historical price, σ is the constant variance of underlying price, and W represents a standard Winner process. From Ito lemma, the logarithmic of underlying price should follow this formula.

$$d\ln S = (\mu - \frac{1}{2}\sigma^2)dt + \sigma dW$$

Therefore, the logarithmic of underlying price follows a normal distribution. However, when applying this theory into practical market options, the normal distribution will express a phenomenon of fat tail at both sides of the distribution. Black and Scholes proposed this models basing on the following assumptions: (1) no dividend before the option maturity; (2) no arbitrage; (3) a constant risk-free interest rate; (4) no transaction cost or taxes; (5) divisible securities; (6) continuous trading; (7) constant volatility. For a given time during the trading day, if the market releases the option price, the option volatility can be calculated by inverting the Black-Scholes formula.

On one hand, if time to maturity is fixed, volatility smile is defined as a curve that the implied volatility changes with different strikes. In a long-observed pattern, the volatility smile looks like a smile. The implied volatility of at-the-money option is smaller than that of in- or out-the-money option. When the implied volatility of out-the-money put option is larger than that of out-the-money call option, this curve is called implied volatility skew. In Zhang and Xiang (2008), they defined the concept of moneyness as the logarithm of the strike price over the forward price, and normalized by standard deviation of expected asset return as follows.

$$\xi \equiv \frac{\ln\left(K / F_0\right)}{\bar{\sigma}\sqrt{\tau}}$$

where $\bar{\sigma}$ is the historical volatility of underlying asset price; τ is the time to maturity; K is strike price; F is the forward index level. Then, implied volatility smirk is defined as employing the moneyness as independent variable. Implied volatility is changed according to different moneyness.

On the other hand, if the implied volatilities with different strikes and a given maturity are combined together under a certain weighted scheme, then the implied volatility term structure can be defined as a curve that weight implied volatilities change with different maturities. When the weight implied volatilities of call options and put options are calculated respectively, we will obtain a call implied volatility curve and a put implied volatility curve. The spread between call and put curve is called call-put term structure spread. This spread contains certain market information.

When the implied volatility of put term structure is larger than that of call term structure, this means market participants worry about the market at the maturity date. If these two curve cross, there are two conditions. First, for the case of implied volatility of put curve larger than call curve before the cross point and smaller than call curve after the cross point, it means the market trend may reverse in a short term and go up. Second, for the case of implied volatility of call curve larger than put curve before the cross point and smaller put curve after the cross point, this means the market trend may reverse in a short term and go down. Finally, when the implied volatility of call term structure is larger than that of put term structure, the view from investors is that the market still goes up. By using this functionality of term structure, we can employ the term structure to predict the underlying asset price efficiently, which is discussed in more detail in chapter 2 and chapter 9.

(2) Local Volatility Model

To forecast the underlying asset price, local volatility also has a good performance. Local volatility is an instantaneous volatility that is a function of time *t* and underlying asset price S_t. Typically, Dupire (1994) presented a deterministic equation to calculate the local volatility from option price basing on the assumption that all call options with different strikes and maturities should be priced in a consistent manner.

$$\sigma(K,T) = \sqrt{2\left(\frac{\frac{\partial C}{\partial T} + rK\frac{\partial C}{\partial K}}{K^2\frac{\partial^2 C}{\partial K^2}}\right)}$$

Nevertheless, this deterministic function suffers two weak points. First, because local volatility is a function of both strike and time to maturity and it is possible that not all the same list of strikes are available at each time to maturity, the number of local volatilities is finite and usually is not enough for further calculation and applications. As a result, researchers are inclined to use interpolation method to obtain a series data of local volatility for further calculation. In this way, the algorithm of interpolation becomes very important because a weak algorithm will create a great problem to the precision. Second, the shortcoming is the intrinsic problem of the Dupire's equation. The indeterminacy of the equation may cause the local volatility to be extremely large or tremendous small. In chapter 6, we propose to use mean-reversion process to overcome these defaults and improve the model. As local volatility is a function of time and underlying asset price, it owns the ability of predicting underlying asset price if we can construct the local volatility surface.

(3) Stochastic Volatility Model

In stochastic volatility model, the volatility is considered as a stochastic process. The stochastic volatility model assumes that the underlying asset price follows a Geometric Brownian process. In the Black-Scholes model, the volatility is assumed to be constant over the period of time to maturity. However, this can explain the phenomenon that the volatility smile and skew vary with different strikes. The stochastic volatility model can solve this problem. Typically, Heston (1993) proposed a Heston model, which considers the underlying asset price process and volatility process as random processes. And these two processes have a constant correlation.

The underlying level process of Heston model is composed of two terms, the price drift term and the volatility with random motion term. The volatility process is also composed of two terms, a volatility drift with mean reversion functionality and a volatility of volatility with random motion. The model is formulated as follows.

$$dS_t = \mu(t)S_t dt + \sqrt{V_t}S_t dW_1$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma\sqrt{V_t}dW_2$$

where θ is the long-run mean level of volatility, κ is the speed of instant volatility returning to long-run mean level, σ is volatility of volatility. These three parameters satisfy the condition of $2\kappa_t\theta > \sigma^2$ and ensure the process of V_t to be strictly positive. Besides, W_1 and W_2 are two standard Wiener process and have a correlation of ρ.

The Heston model is widely applied. This model is used in equity market, gold market, and foreign exchange market. Besides, many extended models are proposed based on Heston model. Christoffersen et al. (2006) proposed a two-factor stochastic volatility model basing on the Heston Model to control the level and slope of the volatility smirk. Andersen et al. (2002) and Chernov and Ghysels (2000) employed an Efficient Method of Moments approach to estimate the structural parameters of Heston model. Bates (2000) used an iterative two-procedure to measure the structural parameters and spot volatilities.

As Heston model simulates the relationship between implied volatility and underlying asset price, this model also owns the ability of forecasting underlying asset price. In chapter 3, we use the Heston model to predict HSI by considering the correlation of underlying level and volatility process as dynamic variable. The underlying asset price is simulated by the analytic solution of Geometric Brownian motion.

Chapter 2 A Novel Model-free Term Structure for Stock Prediction

Implied volatility term structure contains market views. This chapter firstly calculates model-free implied volatility term structure. Then, this term structure is applied as the long-run mean level of Heston model. Since Heston model assume both underlying asset price process and volatility process as stochastic process, the Geometric Brownian motion is used to forecast underlying asset price.

2.1 Introduction

2.1.1 Background

Volatility term structure represents that the implied volatility varies with different times to maturities. While analyzing implied volatility term structure, the key point is to figure out how the implied volatility is calculated from options market data. There are mainly two types of implied volatility valuation methods: the model-based implied volatility and the model-free implied volatility.

The most widely used model-based implied volatility valuation method is the Black-Scholes (BS) model. Researchers reverse the BS model and obtain a deterministic volatility function. However, this calculation method suffers a number of constraints. Heston (1993) proposed a stochastic volatility model which assumes volatility is a stochastic process. Secondly, in the field of model-free volatility, Carr and Madan (1998), Demeterfi et al. (1999), Britten-Jones and Neuberger (2000) and Carr and Wu (2009) have presented a volatility expectation based on variance swap contracts. Britten-Jones and Neuberger (2000) further proposed an integrated volatility defined as the integral of call option price and put option price on all strikes at a given expiry.

2.1.2 Motivation

Extant literature has showed that the BS model has a few shortcomings. First, for a given maturity, options with different strikes have different implied volatilities. This is the reason why the BS model can not explain the volatility smile curve. Second, BS model assumes implied volatility at a given maturity to be constant, which is unsuitable for predicting the underlying asset price trend. Hence, in this chapter, we prefer the model-free integrated implied volatility model but use a discrete form of the integral and consider the process risk-neutral.

Besides, the Heston model considers the process of the underlying price as Geometrical Brownian movement with volatility as a stochastic process. We apply the Heston model, as well as Constant Elasticity of Variance (CEV) Model, to establish a deterministic relationship between the underlying price and volatility.

2.2 Volatility model

The Heston model is more complicated than the CEV model. It takes the mean-reversion term into consideration and the assumption is that the two processes of underlying price and volatility are stochastic processes with a constant correlation to each other. The original Heston model is defined as follows.

$$dS_t = \mu(t) S_t dt + \sqrt{V_t} S_t dW_1$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma\sqrt{V_t} dW_2$$

$$\rho = \langle W_1, W_2 \rangle \tag{2.1}$$

where θ is the long-run mean level of volatility, κ is the speed of instant volatility returning to long-run mean level, σ is volatility of volatility. These three parameters satisfy the condition of $2\kappa_t\theta > \sigma^2$ and ensure the process of V_t to be strictly positive. Besides, W_1 and W_2 are two standard Wiener process and have a correlation of ρ.

2.3 Model-Free Term Structure

2.3.1 Model-Free Implied Volatility

Taylor et al. (2010) define a model-free implied volatility which is an expected implied volatility integrated from different option strikes by option prices. They compare this model-free implied volatility with at-the-money implied volatility and realized volatility in respect of volatility information content. They find that the model-free implied volatility outperforms the other volatility model and is more informative. Similarly, Carr and Wu (2009) defined a risk-neutral integrated volatility, marked as $E_t^Q\left[IV_{t,T}\right]$, over a given period [$t$,$T$] through the options with different strike prices.

$$E_t^Q\left[IV_{t,T}\right]=\frac{2}{T-t}\int_0^\infty \frac{Q_t(K,T)}{P_t(T)K^2}dK \tag{2.2}$$

where $P_t(T)$ is the value of a bond at time t, $Q_t(K,T)$ is the call option price at time t with strike price K and maturity T when strike price K is greater than the underlying asset price or the put option price at time t with strike price K and maturity T when strike price K is less than the underlying asset price.

However, this is a continuum formula for model-free implied volatility. As we focus on the Hong Kong options market, we use a discrete version of Equation (2.2), as follows.

$$\sigma_t^2=\frac{2}{T-t}\sum_i^N\frac{\Delta K_i}{K_i^2}e^{r(T-t)}Q_t(K,T)-\frac{1}{T-t}\left[\frac{F}{K_0}-1\right]^2 \tag{2.3}$$

where N is the number of options for a given maturity T at time t, F is the forward price. According to the Hang Seng Index (HSI), forward price is defined as the forward index level of HSI, which is calculated as follows.

$$F=K+e^{r(T-t)}*(C_K-P_K) \tag{2.4}$$

where K is the strike price of out-the-money call or put options. C_K is the out-the-money call option price. P_K is the out-the-money put option price.

Since the model-free implied volatility is calculated on a given maturity with different strikes, we can draw an implied volatility term structure with different times to maturities, though for a given expiry, implied volatility is constant through the option lifetime. This is not really true in real markets. Therefore, we propose a cubic function to fit the implied volatility term structure. The spirit of the cubic function is similar to the least squares method.

We define cubic function $\varphi(x)$ as follows. The left hand side of the function represents the fitted implied volatility.

$$\varphi(x)=a_0+a_1x+a_2x^2+a_3x^3=\sum_{i=0}^m a_ix^i \tag{2.5}$$

where m=3 is the largest power of the variable, x represents time to maturity (measured in years). a is the corresponding parameters. Our aim is to find an optimal parameter vector a such that the difference between fitted implied volatility and real implied volatility is minimal. We define the difference δ_j to be the result of real data minus fitted data.

$$\delta_j = y_j - \varphi_m(x_j) \tag{2.6}$$

where *j=1,…,n, n* means the total number of real implied volatility, y_j is the real implied volatility, and $\varphi_m(x_j)$ is the estimated implied volatility at *j*th data point. It is hard to get an optimal vector only by comparing this difference because the difference may be small when the real data fluctuate dramatically through a constant mean value. Hence, we use a squared error method to compare the difference.

$$F(a_0, a_1, ..., a_m) = \sum_{j=1}^{n} \delta_j^2 = \sum_{j=1}^{n} \left[y_j - \varphi_m(x_j) \right]^2 \tag{2.7}$$

where $F(a_0, a_1, ..., a_m)$ represents the summation of fitting errors. Obviously, if we minimize the summation of errors, we will obtain an optimal parameter vector for the cubic function. Since the parabolic function will also have an optimal value at a given variable range, we differentiate (2.7) and get a partial differential equation of summation error function to a given parameter.

$$\frac{\partial F}{\partial a_i} = -2\sum_{j=1}^{n} \left[y_j - \sum_{i=0}^{m} a_i x_j^i \right] x_j^i = 0 \tag{2.8}$$

We let the partial differential result to be zero. In this way, the optimal value of the parabolic function is derived. By moving the real implied volatility to right hand side, (2.8) is transformed into (2.9).

$$\sum_{i=0}^{m} a_i \left(\sum_{j=1}^{n} x_j^{i+j} \right) = \sum_{i=1}^{n} y_j x_j^i \tag{2.9}$$

There are four parameters in the cubic function. Hence, *i=0,1,2,3*. At least four pairs of real data points are needed to compute the optimal parameter vector.

2.3.2 Price Distribution of Underlying Asset

By using the cubic parameter estimation, we can fit the curve of implied volatility term structure from real market data. In this section, this curve is used for stock price distribution analysis. But before we move on, it is necessary to return to the original Heston model, on which the stock price distribution is based.

By using historical options data, we are able to estimate the optimal parameters in Heston model. Given these parameters, stochastic processes of underlying asset price and volatility are derived from the Heston model. However, since the process of volatility is calculated from historical options data, which do not reflect the current or future market view of investors, the process of underlying price calculated by the volatility process also lacks future market information. Therefore, the implied volatility term structure described above is able to solve this problem because model-free term structure contains investors' views about what future options market looks like. The process of volatility is generated from the Heston model by using the implied volatility term structure as the long run mean level. To take full advantage of implied volatility term structure, we generate the underlying price process, based on it.

In the basic Heston model, the underlying asset price is determined by a stochastic process with constant drift rate and stochastic volatility. Stochastic volatility is another stochastic process but has a correlation with the first process. Therefore, the underlying process is very similar to the geometric Brownian process (GBP). In GBP, the underlying asset price process is a stochastic process with constant drift rate and constant volatility. Stochastic differential equation (SDE) of GBP is shown as follows.

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{2.10}$$

where μ is the drift rate in percent. Basically, the drift rate is equal to the risk-free interest rate minus the stock dividend. σ is percent volatility. They are both assumed to be constant in the assumption of Geometric Brownian movement. W_t is a standard Wiener process.

In our model, we have already fitted out the implied volatility term structure. The term structure of interest rate is also available from HIBOR[1]. For a given time *t* and initial stock price S_0, an analytical solution can be derived from the SDE of (2.10) by applying the Ito's interpretation[2] (Ito (1951)).

$$S_{t+1} = S_t \exp\left(\left(\mu_t - \frac{V_t}{2}\right)t + \sqrt{V_t} W_t^S\right) \tag{2.11}$$

where S_t is the stock price at time *t*. Right hand side of (2.11) represents calculation of future stock price based on the initial value. For a given time *t*, the implied volatility is obtained from volatility term structure described above. μ_t is the drift rate, a function of time *t*. According to Steven Heston (1993), drift rate of the stochastic process is equal to the interest rate minus the stock dividend, namely $\mu_t = r_t - D_t$. W_t^S is a Wiener process, which is generated by normal distribution with mean of 0 and variance of 1. However, since W_t^S has an optimal correlation with W_t^V, they are calculated as follows.

---

[1] HIBOR is Hong Kong Interbank Offered Rate, the annualized offer rate that banks in Hong Kong offer for a specified period ranging from overnight to one year, normally including overnight, 1 week, 2 weeks, 1 month, 2 months, 3 months, 6 months, and 1 year.

[2] In mathematics, a particular type of stochastic process applies the Ito's Lemma to calculate the differential of a function. It is named after Kiyoshi Ito, who discovered the lemma. It is the counterpart of stochastic calculus in ordinary calculus. It uses the Taylor series expansion by conserving the second order term in calculus. This lemma is widely used in finance and related fields. It is best known for its application for deriving the Black-Scholes option pricing model.

$$W_t^V = \sqrt{dt}\varepsilon$$

$$W_t^S = \rho W_t^V + \sqrt{1-\rho^2}\sqrt{dt}\varepsilon \tag{2.12}$$

Similarly, the stochastic process of implied volatility based on term structure is generated as follows.

$$V_{t+1} = V_t e^{\left(\kappa(\theta - V_t) - \frac{\sigma^2}{2}\right)dt/V_t + \frac{\sigma W_t^V}{\sqrt{V_t}}} \tag{2.13}$$

where κ is the mean reversion speed; θ is the long run mean level of implied volatility which can be obtained from term structure; σ is volatility of volatility.

2.4 Empirical Tests

2.4.1 CEV model

The constant elasticity of variance (CEV) option price model was first proposed by Cox (1975), which contains the Black-Scholes model as a member. Cox assumes that the underlying price process is a stochastic process driven by a stochastic differential equation.

$$dS_t = \mu S_t dt + \sigma S_t^\gamma dW_t \tag{2.14}$$

where μ, σ and γ are constant parameters. μ is the drift rate. Here, we assume it to be risk-free interest rate. σ is volatility of volatility. γ is the constant parameter. They satisfy conditions of $\sigma \geq 0$ and $\gamma \geq 0$. At any given instant time *t*, the standard deviation of returns is defined as follows.

$$V_t = \sigma S_t^{(\gamma-2)/2} \tag{2.15}$$

Therefore, the “CEV exponent” (Emanuel and MacBeth，1982) γ determines the relationship between volatility and the underlying price. When γ equals to 2, volatility equals to the constant variable σ. This is the standard BS model and the option price in CEV model is the same as that in the BS model. When γ is less than 2, we can see the leverage effect from (2.15), which means the underlying asset price increases when volatility decreases. In contrast, when γ is greater than 2, this is the so-called inverse leverage effect, where volatility rises with increase in the underlying price.

Based on the stochastic differential Equation (2.14), we can draw the distribution of the underlying price from stochastic volatility as follows (according to the Geometry Brownian Motion, GBM).

$$S_{t+1} = S_t e^{(r_t - \frac{1}{2}(\frac{V_t}{S_t^{1-\gamma}})^2)t + \frac{V_t \sqrt{t} W_t^S}{S_t^{1-\gamma}}} \tag{2.16}$$

where r_t is the risk-free interest rate.

In this section, we combine (2.15) and (2.16) and use the Monte Carlo simulation to calculate distribution of the underlying asset price. Given the initial underlying price S_0, we can calculate the initial volatility by applying (2.14). Iteratively, the next day price is computed from volatility of the previous day by using (2.16).

2.4.2 Empirical Analysis

(1) Data Description

For our empirical test, we choose the options market data from Hang Seng Index options. Although the original file contains transaction data of every minute, we select only the closing price of every day as the option price. The options data span from 2 January 2007 to 31 December 2009, covering 739 trading days and 321,932 trading records. Each record contains contract information of the option, the trading time (precise to the second), bid and ask price at the trading time, risk-free interest rate and closing price of HSI. At the same time, we also compute simple averages of bid and ask price as the closing price of the option. Since the strike price and time to maturity are contained in the option contract, we can use the BS model to calculate the implied volatility from each entry data. To make the comparison convenient, we append the implied volatility at the end of each entry.

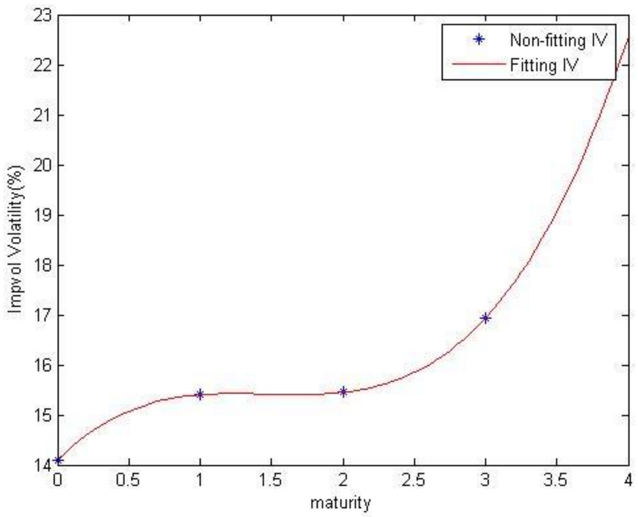

Fig. 2-1 Model-Free Term Structure Fitting Curve.

As the implied volatility of options with different maturities and different strike prices can generate a surface, we calculate the implied volatility term structure from it by averaging the implied volatility along the strike axle. Besides, we only consider options with short maturities of less than six months. Fig. 2-1 shows the average volatility in the future, for 4 months. The stars represent the implied volatility data before fitting. The red line is the model-free term structure curve after fitting. This fitting curve functions as an average implied volatility for Heston model simulation. There are twenty-one trading days each month. Then, there are twenty-one points between each maturity.

(2) Heston and CEV simulation

As mentioned in Section 2.4.1, CEV model contains μ, σ and γ constant parameters, in which μ is equal to interest rate as we assume there are no dividends during our testing period. σ is volatility of volatility. Therefore, we calculate σ from the time series and consider standard deviation of the term structure as σ. Then, there will be an unique sigma each day. According to Ncube (2009), value of γ is important because γ determines the relationship between volatility and the underlying price. Similarly, we set γ equals to 0.5.

We estimate the parameters of Heston model using the historical market options data. The optimal parameters are set to be as follows.

$$\hat{\kappa} = 1.746, \hat{\theta} = 0.306, \hat{\sigma} = 0.499, \hat{\rho} = \text{-}0.899$$

We simulate the Heston volatility process by using the term structure as the long-run mean level of volatility. The above optimal parameters are also used in simulation of volatility process. When simulating the underlying price stochastic process, we fix the random seed of Geometric Brownian motion solution to be 11.

Then, the random motion term can be calculated from these random numbers. The random motion term of volatility process is calculated from that of underlying asset price process by using the correlation coefficient above. Based on all these parameters, we generate the process as shown in Fig. 2-2.

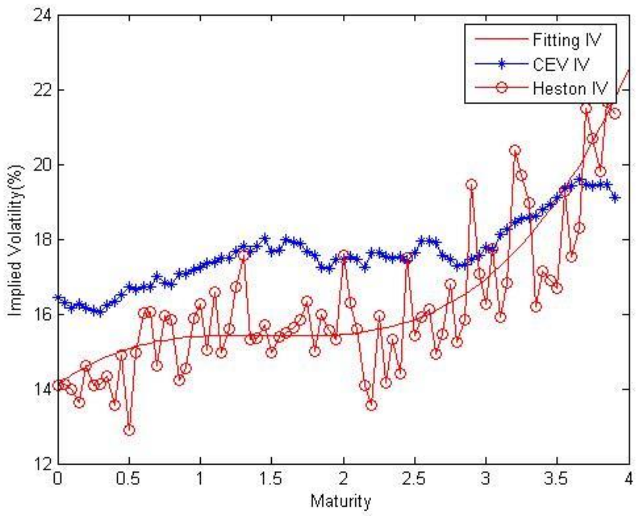

Fig. 2-2 Heston and CEV IV Process.

The red cycle line represents the mode-free fitting term structure. The blue star line is the stochastic process of CEV volatility. Due to the small value of γ, width of vibration of volatility is small. The red cycle line is the Heston volatility process, generated by using the Monte Carlo simulation. After applying implied volatility term structure as long-run mean level, the volatility vibrates along the fitting term structure curve.

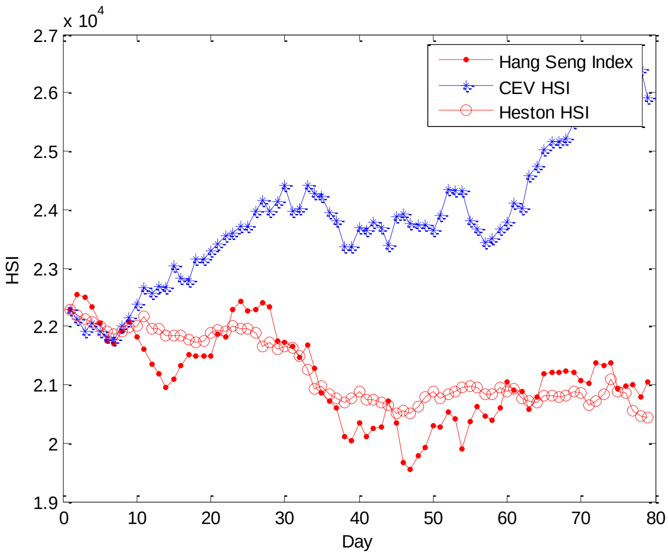

Fig. 2-3 Heston and CEV HSI Process vs Hang Seng Index.

As described in Section 2.3.2, the underlying price process is also a stochastic process and has a correlation with the volatility process. The optimal correlation coefficient is -0.899. By using the result of Fig. 2-2, we simulate the process of the underlying price as shown in Fig. 2-3.

In Fig. 2-3, the red cycle line is the time series of Hang Seng Index from 2 December 2009 to 26 March 2010. The blue star line is the CEV underlying process of HSI. The red cycle line is the Heston underlying process of HSI. Since the process of underlying price has a high correlation with volatility process, the underlying series also vibrates along the time series of HSI.

Table. 2-1 One-Day Ahead Comparison.

Model	Distribution Errors	Trend Prediction (%)	One-Day-Ahead Errors
CEV	2810.8	56.41	1393.35
Heston	375.90	62.82	211.92

To evaluate the prediction ability, we compare the one-day-ahead prediction errors of CEV model and Heston model. The error is measured by Mean Absolute Error (MAE) in (2.17).

$$E = \frac{1}{N}\sum_{i=1}^{N}\left|\hat{y}_i - y_i\right| \tag{2.17}$$

where $\hat{y}_i$ is the *i*th estimated underlying price; y_i is the *i*th market HSI.

As shown Table. 2-1, average errors of Heston model are smaller than CEV model, which means our Heston model with adaptive correlation coefficient scheme works better than the CEV model in respect of one-day-ahead prediction of the underlying price. Besides, we also estimate the distribution performance of our model. The underlying price distribution is a measure of future price based on the term structure. In our test, we measure it over a horizon of 79 days of price distribution. The MAE error of our model is nearly three times less than that of CEV model. Except the position comparison of underlying price, we also compare the precision of trend prediction of our model with CEV model. As shown in Table. 2-1, our model has 62.82% of the trend prediction precision over the 79-days period while the CEV model has only 56.41%.

2.5 Conclusions

Prediction of underlying price has always been a hot and interesting topic in academic research on financial markets. Different from the traditional time series method, we estimate distribution of the underlying price in a different way, using a model-free implied volatility term structure. We first introduce the stochastic model in Section 2 and find that stochastic volatility is most suitable for prediction of the underlying price. Before simulating distribution of the underlying price, we calculate the model-free implied volatility term structure. Finally, we simulate distribution of the underlying price based on the term structure. Our results are better than the CEV model in terms of one-day-ahead prediction performance and the 79-days distribution of underlying price.

Chapter 3 An Adaptive Correlation Heston Model for Stock Prediction

The traditional Heston model assumes that the correlation between underlying asset price process and volatility process is constant. This chapter investigates this correlation and proposes an adaptive correlation coefficient scheme to estimate the Heston parameters. The performance of adaptive correlation Heston model is compared with traditional Heston model on prediction precision.

3.1 Introduction

3.1.1 Background

Heston (1993) proposed a stochastic model for option pricing, the Heston Model, which considers the process of the underlying price as a Geometrical Brownian movement with volatility as a stochastic process. The process of volatility takes the long-run mean of volatility into consideration, which ensures that volatility varies within a reasonable range. This overcomes disadvantages of the Black-Scholes model, which is constrained to several assumptions. Besides, Heston assumed that there is a constant correlation between underlying price process and stochastic volatility process. This makes sure the process of underlying price evolves reasonably.

However, after observation of market data of Hang Seng Index (HSI), we find that the correlation coefficient is not necessarily a constant. It varies from time to time. Coefficients of volatility process, including rate of reverting to mean level, the long-run mean volatility level and the volatility of volatility, vary time-dependently, which ensures the process of volatility evolves within a corrective process. Therefore, we apply the Heston model with dynamic correlation coefficient to establish a deterministic relationship between the underlying price and volatility.

3.1.2 Motivations

We find that the constant assumption does not hold in HSI market. Fig. 3-1 shows comparison of VHSI and HSI from June 2008 to July 2011. The red line represents the time series data of HSI. The blue line represents the time series of VHSI. The two are obviously highly correlated. When HSI went down during the period between June 2008 and November 2008, the trend of VHSI was obviously upward. When HSI climbed from March 2009 to Nov 2009, VHSI also declined gradually. When the price fluctuated from January 2010 to July 2011, VHSI also experienced high fluctuations.

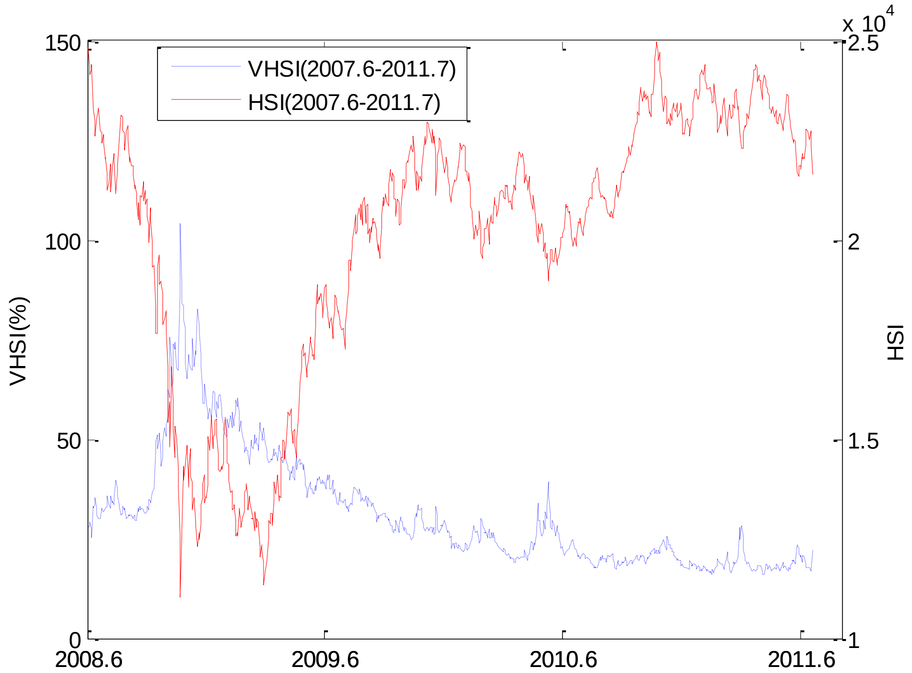

Fig. 3-1 Time Series of VHSI and HIS.

To numerically analyze the correlation between VHSI and HSI, we use the Pearson's correlation coefficient which defines the correlation as the covariance of two variables divided by the product of their standard deviation.

$$\rho_{X,Y} = \frac{\mathrm{cov}(X,Y)}{\sigma_X \sigma_Y} = \frac{E\left[(X-\mu_X)(Y-\mu_Y)\right]}{\sigma_X \sigma_Y} \tag{3.1}$$

Table. 3-1 Correlation Coefficients between VHSI and HIS.

Section No.	Time Period	Correlation Coefficient	Number of Observations
1	Jun 2008 ~ Nov 2008	-0.9629	120
2	Dec 2008 ~ Mar 2009	0.0974	80
3	Apr 2009 ~ Nov 2009	-0.8858	160
4	Dec 2009 ~ Jul 2011	-0.5937	409

Here, in our analysis, *X* represents the series of VHSI, *Y* represents the series of HSI. We divide the time series data into four sections and calculate their respective correlation coefficients. Table. 3-1 shows that different sections have different time periods and the correlation coefficients are also different from each other. Besides, the number of observations affects the correlation coefficients.

Correlation coefficients of Sections 1 and 3 are relatively high. Their numbers of observations are also low. Section 4 also has a correlation of -0.5937 but its number of observations is high. As is known, observations over a long period of time may be unable to capture the trend of time series. Nevertheless, Section 2 has observations over a short period but its correlation is also low. That is because the trend of this section is fluctuating and unstable. The above analysis suggests that the correlation coefficient may be affected by two factors:

(1) The number of observations affects the correlation coefficient. If the observation period is too large, the correlation coefficient is affected severely.
(2) The trend of the underlying asset price should be monotonous. If the process of underlying asset price fluctuates during the observation period, the coefficient also changes dramatically.

Therefore, a suitable length of time period of observations and a stable time series trend are critical for obtaining high correlation coefficient. In the next section, we propose an adaptive correlation coefficient scheme to capture the correlation between the underlying price process and the volatility process.

3.2 Adaptive Correlation Coefficient Model

3.2.1 Adaptive Correlation Heston Model

Compared with the Black-Scholes model, Heston (1993) assumes a European option with strike price K and time to maturity T can be priced using the following equation. This is a closed-form solution for the Heston model (Moodley, 2005).

$$C(S_t, V_t, t, T) = S_t P_1 - K e^{-r(T-t)} P_2 \tag{3.2}$$

where

$$P_j(x, V_t, T, K) = \frac{1}{2} + \frac{1}{\pi} \int_0^{\infty} \mathrm{Re}\left(\frac{e^{-i\varphi \ln(K)} f_j(x, V_t, T, \varphi)}{i\varphi} \right) d\varphi$$

$$x = \ln(S_t)$$

$$f_j(x, V_t, T, \varphi) = \exp\left\{ C(T-t, \varphi) + D(T-t, \varphi) V_t + i\varphi x \right\}$$

$$C(T-t, \varphi) = r\varphi i T + \frac{a}{\sigma^2}\left[\left(b_j - \rho\sigma\varphi i + d \right)\tau - 2\ln\left(\frac{1 - g e^{dr}}{1 - g} \right) \right]$$

$$D(T-t,\varphi)=\frac{b_j-\rho\sigma\varphi i+d}{\sigma^2}\left(\frac{1-e^{dr}}{1-ge^{dr}}\right)$$

$$g=\frac{b_j-\rho\sigma\varphi i+d}{b_j-\rho\sigma\varphi i-d}$$

$$d=\sqrt{\left(\rho\sigma\varphi i-b_j\right)^2-\sigma^2\left(2u_j\varphi i-\varphi^2\right)}$$

For $j=1,2$, we have

$$u_1=\frac{1}{2},\ u_2=-\frac{1}{2},\ a=\kappa\theta,\ b_1=\kappa+\lambda-\rho\sigma,\ b_2=\kappa+\lambda$$

Thus, Heston's method seems to be suitable for pricing of European options. However, as mentioned above, we observe that the correlation coefficient between the underlying asset price and volatility is not constant. It varies with time and shape of stochastic process. Therefore, we propose to use the principle of least squares method to estimate the correlation coefficient.

From the market option data, we already know the option prices. Then, we can use (3.2) to estimate the option price and compare the results with real market data. We rewrite the function of (3.2) as follows.

$$C(S_t,V_t,t,T)=F\left(S_t,V_t,t,T,K;\kappa,\theta,\sigma,\rho\right) \tag{3.3}$$

where parameters of S_t,V_t,t,T,K are already known, or can be calculated from real options market data. Here, we define a vector $\alpha=[S_t,V_t,t,T,K]$. The parameters of $\kappa,\theta,\sigma,\rho$ need to be estimated. We define a parameter vector $\beta=[\kappa,\theta,\sigma,\rho]$. The difference between the estimated option price and the real option price is $r_i=y_i-F\left(\alpha_i,\beta\right)$, where y_i is the ith real option price. The squared summation of the difference is Φ.

$$\Phi=\sum_{i=1}^{n}r_i^2=\sum_{i=1}^{n}\left(y_i-F\left(\alpha_i,\beta\right)\right)^2 \tag{3.4}$$

where n is the total number of options data. According to the spirit of the least squares method, if the first differential of (3.4) is equal to 0, Φ will arrive at an optimal value. That is

$$\frac{\partial\Phi}{\partial\beta_j}=2\sum_{i=1}^{n}r_i\frac{\partial r_i}{\partial\beta_j}=-2\sum_{i=1}^{n}r_i\frac{F(\alpha_i,\beta)}{\partial\beta_j}=0 \tag{3.5}$$

where j = 1,2,3,4 corresponds to the parameter index of β. By solving equation (3.5), we finally obtain an optimal parameter group, including the optimal correlation coefficient.

3.2.2 Distribution of the Underlying Asset Price

In Section 3.2.1, we propose an adaptive correlation coefficient scheme in the Heston model. By using historical options data, we are able to estimate the optimal parameters in the Heston model. Given these parameters, stochastic processes of the underlying asset price and volatility are specified from the Heston model. However, since the process of volatility is calculated from historical options data, which does not reflect the current or future market view of investors, the process of the underlying price calculated by the volatility process also lacks future market information. Therefore, we use the volatility term structure as the long-run mean level of the Heston model to solve this problem because the volatility term structure incorporates market view of future options market. Then, the underlying price process is calculated from this volatility process.

In the basic Heston model, the underlying asset price is determined by a stochastic process with a constant drift rate and stochastic volatility. Stochastic volatility is another stochastic process but has a correlation with the first process. Therefore, the underlying process is very similar to the geometric Brownian process (GBP). In GBP, the underlying asset price process is a stochastic process with a constant drift rate and constant volatility. Stochastic differential equation (SDE) of GBP is as follows.

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{3.6}$$

where μ is the percentage drift rate. Basically, the drift rate is equal to the risk-free interest rate minus the stock dividend. σ is percent volatility. They are both assumed to be constant in the assumption of geometric Brownian movement. W_t is a standard Wiener process.

` In our model, we have already incorporated the implied volatility term structure. The term structure of interest rate is also available from HIBOR[3]. For a given time *t* and initial stock price S_0, an analytic solution can be derived from SDE of (3.6) by applying the Ito's interpretation[4] (Ito, 1951).

$$S_{t+1} = S_t \exp\left(\left(\mu_t - \frac{V_t}{2}\right)t + \sqrt{V_t}W_t^S\right) \tag{3.7}$$

---

[3] HIBOR is Hong Kong Interbank Offered Rate, the annualized offer rate that banks in Hong Kong offer for a specified period ranging from overnight to one year, normally including overnight, 1 week, 2 weeks, 1 month, 2 months, 3 months, 6 months and 1 year.

[4] In mathematics, a particular type of stochastic process applies the Ito's Lemma to calculate the differential of a function. It is named after Kiyoshi Ito, who discovered the lemma. It is the counterpart of stochastic calculus in ordinary calculus. It uses the Taylor series expansion by conserving the second order term in calculus. This lemma is widely used in finance and related fields. It is best known for its application for deriving the Black-Scholes option pricing model.

where S_t is the stock price at time t. Right hand side of (3.7) represents the calculation of future stock price based on the initial value. For a given time t, the implied volatility is obtained from the volatility term structure described above. μ_t is the drift rate, a function of time t. According to Heston (1993), drift rate of the stochastic process is equal to the interest rate minus the stock market dividend, namely $\mu_t = r_t - D_t$. W_t^S is a Wiener process generated by normal distribution with mean of 0 and variance of 1. However, since W_t^S has an optimal correlation with W_t^V, they are calculated as follows.

$$W_t^V = \sqrt{dt}\varepsilon$$

$$W_t^S = \rho_t W_t^V + \sqrt{1-\rho_t^2}\sqrt{dt}\varepsilon \tag{3.8}$$

where ρ_t is the dynamic correlation coefficient of Heston model. Similarly, stochastic process of implied volatility based on term structure is generated as follows.

$$V_{t+1} = V_t e^{\left(\kappa(\theta - V_t) - \frac{\sigma^2}{2}\right)dt/V_t + \frac{\sigma W_t^V}{\sqrt{V_t}}} \tag{3.9}$$

where κ is the mean reversion speed; θ is the long run mean level of implied volatility, which can be obtain from the term structure; σ is volatility of volatility.

3.3 Empirical Tests

3.3.1 CEV model

The constant elasticity of variance (CEV) option price model was proposed by Cox (1975). It includes the Black-Scholes model as a member. Cox assumed that the underlying price process is a stochastic process driven by a stochastic differential equation.

$$dS_t = \mu S_t dt + \sigma S_t^\gamma dW_t \tag{3.10}$$

where μ, σ and γ are constant parameters, satisfying conditions of $\sigma \geq 0$ and $\gamma \geq 0$. At any given instant time t, standard deviation of returns is defined as follows.

$$V_t = \sigma S_t^{(\gamma-2)/2} \tag{3.11}$$

Therefore, the "CEV exponent" (Emanuel and MacBeth, 1982) γ determines the relationship between volatility and the underlying price. When γ equals to 2, volatility equals to the constant variable σ. This is the standard BS model and the option price in CEV model is the same as in the BS model. When γ is less than 2, we can see the leverage effect from (3.11), which means the underlying price increases while volatility decreases. In contrast, when γ is greater than 2, this is the so-called inverse leverage effect, where volatility rises with increase of the underlying price.

Based on the stochastic differential equation (3.10), we can draw the distribution of the underlying price from stochastic volatility as follows, in accordance with the Geometry Brownian Motion (GBM).

$$S_{t+1} = S_t e^{(r_t - \frac{1}{2}(\frac{V_t}{S_t^{1-\gamma}})^2)t + \frac{V_t \sqrt{t} W_t^S}{S_t^{1-\gamma}}} \tag{3.12}$$

where r_t is the instant interest rate.

In this section, we combine (3.11) and (3.12) and use the Monte Carlo simulation to calculate distribution of the underlying price. Given the initial underlying price S_0, we can calculate the initial volatility by applying (3.11). Iteratively, the next day price is computed from volatility of the previous day, using (3.12).

3.3.2 Empirical Analysis

As mentioned in Section 3.3.1, CEV model contains constant parameters μ, σ and γ in which μ is equal to interest rate since we assume there are no dividends during the testing period. σ is volatility of volatility. Therefore, we calculate σ from the time series and consider the standard deviation of the term structure as σ. Then, there will be a unique sigma each day. According to Ncube (2009), value of γ is important because γ determines the relationship between volatility and the underlying asset price. Similarly, we set γ equals to 0.5.

Before we simulate volatility process of the CEV model, it is important to discretize it. In this simulation, we consider the step of CEV model as one day measured by year. There are twenty-one trading days in a month, which means there are twenty-one volatilities in one month. By using Monte Carlo simulation, we generate the volatility process. Besides, when generating random number for simulation, we set random seed of normal distribution as 11 and calculate random term of underlying asset price process from these random numbers. At each evaluation day, once we use the underlying price to generate the volatility, according to (3.12), we can use this volatility to estimate the underlying price of the next day. The first underlying price is from market data.

The Heston model is a stochastic volatility model. Parameters of the Heston model affect the volatility process greatly. Hence, in order to obtain an optimal group of parameters, we train the Heston model by using historical options data from 2 January 2007 to 30 November 2009. Data for the last month of the market option is left for testing. Section 3.3.2 discussed the four parameters of $\beta = [\kappa, \theta, \sigma, \rho]$ to be estimated. In our empirical running results, we obtain the optimal as follows, using the algorithm described in Section 3.3.2.

$$\hat{\kappa} = 1.746, \hat{\theta} = 0.306, \hat{\sigma} = 0.499, \hat{\rho} = \text{-}0.899$$

Implied volatility is a measure of future average volatility, which reflects the degree of panic on part of investors, about future options and stock market. The volatility term structure, therefore, contains significant information about market view. The Heston simulation based on the term structure, therefore, becomes significant. To make use of the historical options market data, we simulate the Heston volatility process by using the term structure as the long-run mean of volatility. The above optimal parameters are also used in simulation of the volatility process.

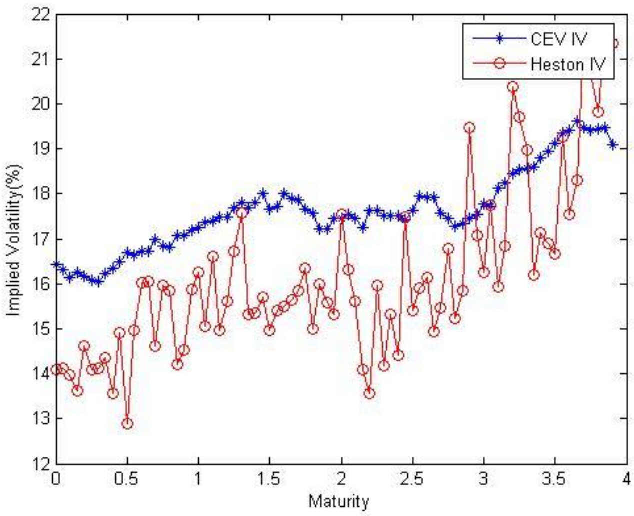

Fig. 3-2 CEV Implied Volatility vs Heston Implied Volatility

As described in Section 3.2.2, the underlying price process is also a stochastic process and has a correlation with the volatility process. The optimal correlation coefficient is -0.899, which is dynamically estimated by least squares method. By using this parameter, we simulate the Heston model and the CEV model. In Fig. 3-1, the blue star line is the CEV implied volatility. The red cycle line is the process of Heston volatility.

Table. 3-2 One-Day Ahead Comparison.

Model	79 Days Errors	One-Day Ahead Errors
CEV Model	2810.8	1393.35
Heston Model	375.90	211.92

As shown is Table. 3-2, average error of the Heston model is smaller than the CEV model, which means our Heston model with adaptive correlation coefficient scheme works better than the CEV model for prediction of one-day-ahead underlying price. Besides, we also estimate the distribution performance of our model. The underlying price distribution is a measure of future price based on the term structure, which we measure over a horizon of 79 days, in the test. The MAE error of our model is nearly three times less than that of the CEV model.

3.4 Conclusions

The Heston model is a stochastic volatility model. It has advantages when we use it to simulate the time series of the underlying price. In This chapter, we first introduce the characteristics of the Heston model and then propose the adaptive correlation coefficient scheme since the constant correlation of the Heston model is unsuitable for HSI and VHSI. In the empirical test, we compare our adaptive correlation scheme with CEV model on both 79 days horizon prediction and one-day-ahead prediction. The results of adaptive correlation Heston model are better than the CEV model in terms of one-day-ahead prediction performance and the 79-day distribution of the underlying price.

Chapter 4 The Algorithm to Control Risk Using Option

Option as a financial derivate performs well in arbitrage. Because of the leverage, it is also a powerful tool for risk management. Option can change the return distribution and to control the risk effectively. However, the complexity of it always keeps investors away from using option to control the risk. This chapter uses basic theories of option and the method of simulation to clarify the significant role option can play in risk management. Different characteristics and effects of different options are analyzed by applying different options into the investment.

4.1 Introduction

Option as a financial derivate can be not only used in arbitrage but also in risk management. It is a derivative financial instrument that specifies a contract between two parties for a future transaction on an asset at a reference price. The buyer of the options gains the right, but not the obligation, to engage in that transaction, while the seller incurs the corresponding obligation to fulfill the transaction.

Because of some special characteristics such as the function of monetary leverage, it can perform well in risk management. Since it was first traded as unified and standardized derivative in CBOE in April 1973, option has developed significantly over the past decades. When Black and Scholes (1973) developed the model to price the option in the paper "The Pricing of Options and Corporate Liabilities", the trading of options started booming. The model is Black-Scholes model.

After decades of development, option is now playing a significant role in financial markets because of its particular characteristics. Though option has many characteristics, such as the leverage and these characteristics, which are good for risk management because of the complexity of option and investor's disregard of risk management, option now is more widely used for arbitrage than risk management. It is well known that options can increase the flexibility of returns available from investment strategies and can truncate the downside risk efficiently. So option as a powerful risk management tool has been widely accepted.

However, few works have focused on this field. Most of investors and scholars put their eyes on the profit which results in risk management ignored. Characteristics of options were first explored by Merton et al. (1978, 1982). After that Bookstaber and Clarke (1983, 1984) developed their theory using Monte Carlo simulation and Capital Asset Pricing Model to analyze the impact of option to change the return distribution. They used CAPM model to state the influence of the option and in their paper they also proved the function of the option in risk management. Both call and put can change the return distribution. Put is useful in truncating the downside risk. The limitation of their papers is that they just illustrated the characteristics of option. They did not prove why the option changed the return distribution and how to choose the right option to control risk. Ahn et al. (1997) solved this problem. Their target was to reduce the VaR by using put option based on cost C. If the cost is limited, they try to find a balance point between the hedge rate and exercise price of put to make the value of VaR the least. This chapter aims to expand the method of risk management and combine both the target of reducing the potential risk and raising the profit by using both put and call option. Because option is powerful in changing the return distribution and making the return flexible, it can provide more choices and extend the efficient frontier for different investors to meet their different risk appetites.

This chapter is composed of four main parts. The first part is the introduction which illustrates the background and previous research in this field. The second part is the theoretical background and the methodology. It can be divided into two parts. The former is the theoretical basis to use the method to analyze risk management with use of option, while the latter part is the model to solve the problem by using option to control the risk and get more profit. The third part discusses the advantages and disadvantages of these two methods. The last part is the conclusion. In this part, the results are analyzed to support risk management.

4.2 Theoretical Background and Model

4.2.1 Theoretical Background

According to surveys by the Wharton School in cooperation with Chase Manhattan Bank and by Ernst and Young (1999), majority of firms have been applying modern financial techniques to manage some of their exposure to factors which can affect the value of assets. Because of the special characteristics, option is widely accepted as a good tool for risk management, especially put option, which can truncate downside risk efficiently. The Black-Scholes model is popular in trading of option. Geometric Brownian motion (GBM) (also known as exponential Brownian motion) is a continuous-time stochastic process in which the logarithm of the randomly varying quantity follows a Brownian motion (Ross (2007)), also called a Wiener process. It is used for simulating the change of asset value and mathematical finance to model stock prices in the Black–Scholes model. The Black–Scholes model is useful for calculating the reference value of the option. The model in This chapter is based on the assumption that the change of the underlying asset's value, S_t, always follows a stochastic process called the Geometric Brownian motion. The equation can be written as following the stochastic differential equation:

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{4.1}$$

where μ and σ are the drift and the diffusion of the asset value, and W_t is a standard Brownian motion. In fact, most of assets can be simulated in this process, whether it is a single asset or a portfolio of assets, such as the Heng Seng Index. The GBM process can better simulate the change in asset value. For any time period t, parameters of μ and σ can be calculated and for the initial value S_t, the analytic solution of Equation (4.1) can be derived based on Ito lemma, which is:

$$S_{t'} = S_t \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)(t'-t) + \sigma W_{t'}\right) \tag{4.2}$$

where $S_{t'}$ is the value of asset in t' after a time step Δt. The conditional distribution of asset $S_{t'}$ is lognormal distribution:

$$\ln S_{t'} \sim N\left[\ln S_t + \left(\mu - \frac{\sigma^2}{2}\right)\Delta t, \sigma^2 \Delta t\right] \tag{4.3}$$

The probability density function of $S_{t'}$ is:

$$f_{S_{t'}}\left(s;\mu,\sigma,\Delta t\right) = \frac{1}{\sqrt{2\pi}} \frac{1}{s\sigma\sqrt{\Delta t}} \exp\left(-\frac{\left(\ln s - \ln S_t - \left(\mu - \frac{\sigma^2}{2}\right)\Delta t\right)^2}{2\sigma^2 \Delta t}\right) \tag{4.4}$$

Equation (4.2) can be used to simulate the change of the asset value. It is a reference for the investor to make decision and can be tested for different investment strategies. Equation (4.4) is the distribution of asset value in time period t' without any risk control. In the next part, this chapter uses different risk management strategies using options. It changes the return distribution efficiently. The model proposed in this chapter is helpful to analyze different risk management strategies quantitatively to help the investors find the proper strategies to control risk. In the following part, both put and call options are taken into consideration to build the model to analyze the different impacts these two kinds of options can have in risk management.

4.2.2 Risk Management Model

Characteristics of change of asset and return distribution have been stated above. The model of risk management using option is built in this part and the model can be divided into two parts. The first part is using put option to control the risk and the second part uses call option. In the end of this part, the two methods are compared to demonstrate the advantages and disadvantages of these two methods.

(1) Risk Management Model using Put Option

In the model where put option is bought to control the risk, an assumption is made that the capital being consumed needs to be less than a fixed cost C. Usually the optimal ratio, h, corresponding to the fixed cost C is less than 1. The first reason is that the main objective is to control risk. If the ratio h is bigger than one, cost in excess of one can be used for higher exercise price option which can affect a larger range of the distribution, which will perform better. The second reason is that if there is no higher exercise price option, the part that is higher than one can be divided and this can be viewed as the arbitrage using option and it can be analyzed singly.

Assume that the strategy is buying option with exercise price, X, at price p_p. The hedging rate can be expressed as

$$h = f\left(p_p\right) \tag{4.5}$$

In reality, the price and the exercise can be obtained from the market, but it is necessary to use the model to find the best balance point as a reference. It is well known that the Black-Scholes model performs well for calculating the price of the option. So the equation using this model can be expressed as:

$$C = hf\left(BS\left(S_t, X, r, \Delta t, \sigma, y\right)\right) \tag{4.6}$$

In this equation, r is the risk free rate and y is annualized, continuously compounded yield of the underlying asset over the life of the option. Equation (4.6) has been solved by Ahn et al. (1997). In reality, the continuous solution is not necessary. Because the number of put options that can be chosen is never too large, combinations of h and X are enumerated if the ratio, h, is viewed as discrete. This is not different from choosing the best strategy by comparing all of these choices. In this chapter, the relationship between h and X is necessary for further research. The return distribution can be divided into two conditions; when using the option with exercise price X. The hedged future value of the underlying asset in time t', $A_{t'}$ can be defined into the format of piecewise function:

$$\begin{cases} A_{t'} = S_{t'} & if \quad S_{t'} \geq X \\ A_{t'} = hX + (1-h)S_{t'} & if \quad S_{t'} < X \end{cases} \tag{4.7}$$

Combining Equations (4.4) and (4.7), the probability density function of the hedged future value of asset, $A_{t'}$, can be defined as:

$$\begin{cases} f\left(A_{t'} \mid S_{t'} \geq X;\mu,\sigma,\Delta t\right)=\frac{1}{\sqrt{2\pi}}\frac{1}{A_{t'}\sigma\sqrt{\Delta t}}\exp\left(-\frac{\left(\ln A_{t'}-\ln S_t-\left(\mu-\frac{\sigma^2}{2}\right)\Delta t\right)^2}{2\sigma^2\Delta t}\right) \\ f\left(A_{t'} \mid S_{t'} < X;\mu,\sigma,\Delta t\right)=\frac{1}{\sqrt{2\pi}}\frac{1-h}{\left(A_{t'}-hX\right)\sigma\sqrt{\Delta t}}\exp\left(-\frac{\left(\ln\left(A_{t'}-hX\right)-\ln\left(1-h\right)-\ln S_t-\left(\mu-\frac{\sigma^2}{2}\right)\Delta t\right)^2}{2\sigma^2\Delta t}\right) \end{cases} \tag{4.8}$$

All parameters in the equation above can be calculated at time period t, except for h and X, which are decision variables. And the relationship between these two parameters can be expressed in Equation (4.6), at a given cost C.

(2) Risk Management Model using Call Option

Option as a good tool of risk management has always been ignored in reality. Some of the works have explored the function of using put option in risk management. It is well known that both call option and put option can change the return distribution effectively. The work done by Bookstaber and Clarke (1983; 1984) also proved that call option also has the function of controlling the risk and raising profit. The Wing Fung Financial Group, for example, earns most of its profit from selling options. It is effective to sell call option to get the premium and to reduce the risk to a certain degree. The same as the illustration above, the model can be built to calculate the distribution of the hedged future value of asset to analyze the function of selling call option in risk management.

The problem still can be divided into two conditions. One is where the option finishes out-of-the-money while the other is it finishes in-the-money. The difference between using call and put option to control risk is that in the call option the hedged ratio h does not relate to cost C because the nature of using call option to control risk is to use the premium on selling call option to resist the risk in a certain degree. It means that once the strategies of investment failed, the premium on selling call option can hedge some of the risk caused by the investment. But the hedged ratio h affects both the conditions of the option that finishes at-the-money and out-of-the-money. In this model, the hedged value of the asset can be defined as:

$$\begin{cases} A_{t'} = hX + \left(1-h\right)S_{t'} + P_c & if \quad S_{t'} \geq X \\ A_{t'} = S_{t'} + P & if \quad S_{t'} < X \end{cases} \tag{4.9}$$

Parameter P_c is the premium earned from selling call option and it can be calculated as

$$P_c = hf'\left(BS\left(S_t, X, r, \Delta t, \sigma, y\right)\right) \tag{4.10}$$

In this model the assumption of h is made the same as the model of put option. Combining Equations (4) and (10), distribution of the hedged future value of asset can be calculated as

$$\begin{cases} f\left(A_{t'} \mid S_{t'} < X;\mu,\sigma,\Delta t\right) = \dfrac{1}{\sqrt{2\pi}} \dfrac{1}{\left(A_{t'} - P_c\right)\sigma\sqrt{\Delta t}} \exp\left(-\dfrac{\left(\ln\left(A_{t'} - P_c\right) - \ln S_t - \left(\mu - \dfrac{\sigma^2}{2}\right)\Delta t\right)^2}{2\sigma^2 \Delta t} \right) \\ f\left(A_{t'} \mid S_{t'} \geq X;\mu,\sigma,\Delta t\right) = \dfrac{1}{\sqrt{2\pi}} \dfrac{1-h}{\left(A_{t'} - hX\right)\sigma\sqrt{\Delta t}} \exp\left(-\dfrac{\left(\ln\left(A_{t'} - hX\right) - \ln\left(1-h\right) - \ln S_t - \left(\mu - \dfrac{\sigma^2}{2}\right)\Delta t\right)^2}{2\sigma^2 \Delta t} \right) \end{cases} \quad (4.11)$$

Combining Equation (4.11) into Equation (4.10), the probability density function of the asset value can be calculated and it can be viewed as the impact of using call option in risk management. But this model is more complicated than the model of put option. Comparing Equation (4.6) and (4.10), it is clear that the parameter of cost C is given as a fixed number, while the P_c in the equation is an independent variable which depends on parameters h and X. In reality, the parameter is in the interval from zero to one, and the exercise price X of call option is also limited. So it is not difficult to find the range of h and X which is suitable.

Up to now, impact of both call and put options can be given mathematically and in detail. From Equations (4.7) and (4.9), we can directly draw the conclusion that options can help improve the value of the underlying asset. That means it has the function of risk management. And furthermore, from Equations (4.8) and (4.11), it is obvious that options can change the return distribution and improve the return in bad cases. Of course, both methods have advantages and disadvantages. This is summarized in the next part.

4.3 Discussion

In this part, advantages and disadvantages of both methods are discussed. By summarizing these, the characteristics of these two methods can be viewed directly and it is helpful to promote the use of options in risk management.

(1) Advantages of method one:

First, using put option in risk management has been accepted widely as it can truncate the downside risk effectively. Especially in extreme cases, it can keep the risk within a range. Second, for any given cost C, it is easy to find the optimal strategy to control risk and it is easy to simulate the impact the strategy can make. Third, the capability of this method is bigger than the method of using call option. Releasing cost C can always keep the risk within an ideal range.

(2) Disadvantages of method one:

This method is just like insurance. The application of this method always leads to a cost. It performs well only in extreme cases. What's more, the impact the method can make is decided by cost C. When the capital that can be used in this method is limited, the impact the method can make is also limited.

(3) Advantages of method two:

While using the call option in risk management, in most cases the investor can not only keep the risk within a certain degree but also can earn excess profit. And this method is not limited by cost. This method performs well in most cases in reality.

(4) Disadvantages of method two:

It is obvious that the boundary of risk in this method is P_c, and if the value of the asset rises to the upper bound, this method reduces the profit. The domain in which the method can perform well is limited, i.e. in extreme cases this method has limited effectiveness. What's more, because of the complexity of the model itself, it is more difficult to find the optimal strategy than method one.

In summary, both methods can change the return distribution well and can extend the efficient frontier to meet the different risk appetites. Both of them can make VaR and CVaR less when applied into the investment.

4.4 Conclusions

This chapter provides a formal analysis of risk management using different kinds of options and builds two models to analyze the different functions of the different methods. By analyzing the role of the option in risk management, the conclusion that can be drawn is that option is a good tool for the investor to use in risk management. It is well known that options can increase flexibility of returns available and this can affect the return distribution directly. What's more, options have the characteristic of leverage. All of these characteristics make option a good tool in risk management.

Equation (4.7) in model one and Equation (4.9) in model two depict that options always help improve the value of the underlying asset in bad cases. That means both put and call options have the function of risk management. And furthermore, Equations (4.8) and (4.11) in model one and two describe the roles of put and call options in risk management briefly. In the third part both advantages and disadvantages of these two methods are discussed, when using them to control risk. The most important meaning is that this is the first paper to state the function of both put and call options in risk management, to build a model to illustrate them and compare the two methods together. Of course there still is a lot of work that needs to be done in the future. It may be possible to use real data to prove these two methods are useful in risk management and use real data to prove that these two methods can perform well to control risk in some aspects.

Chapter 5 Option Strategies: Evaluation Criterion and Optimization

For a specified underlying asset, a large number of options can be chosen, especially for the combined strategies. Compared with stock, designing strategies for option trading for arbitrage is more complex. In reality, most investors design strategies empirically. This chapter uses HSI (Hang Seng Index) as the underlying asset and uses some standard criteria such as VaR, CVaR and so on as references to build the model for design of optimal strategies for investment. A real case is also uses to prove that good results can be obtained, based on a good prediction of the trend of HSI.

5.1 Introduction

Option plays a significant role in financial markets and it is widely used in both arbitrage and risk management. As one kind of financial derivatives, it is an instrument that specifies a contract between two parties for a future sale or purchase of an asset at an agreed price. The buyer of an option gets the right, but not the obligation, to engage in that transaction, while the seller is bound to fulfill the corresponding obligation.

Because of some special features such as the leverage, it can perform well in investment for both arbitrage and risk management. Since it was first traded unified and standardized in CBOE in April 1973, the instrument of option has developed significantly over the past few decades. When Black and Scholes (1973) created the model to price the option in their paper “The Pricing of Options and Corporate Liabilities”, trade in options has been booming. The model is called Black-Scholes model and is still widely used. Most of research works are done by using this model as the foundation. In this chapter, this model is used to calculate the theoretical price of option. In terms of flexibility of using the option in investment and its functions, Bookstaber and Clarke (1983; 1984) have proved that option is powerful in changing the distribution of return. These two papers also point out that the appropriate option strategies can always meet requirements of investors. Apart from the function of changing the return distribution, option itself is a wonderful tool for investment and it is gaining increasing acceptance among investors. Though using option for arbitrage has been very popular and its share in financial markets has been growing, almost all financial organs design their strategies empirically.

In reality, investors trust their experience more than quantitative analysis and so usually draw a trend line based on their experiences, market view and a little data analysis. Then they find options which stay away from the line, which can create space for arbitrage. This method always works if the trend line is accurate but it is limited in some aspects. First, the strategies are not based on quantitative analysis and, therefore, usually they are not the optimal strategies. The fund utilization rate is low and it can ensure benefit maximization only under certain conditions. Second, because these strategies are designed empirically, it is impossible to get hold of the risk level director. The option itself is complex and results in need for constant changes in strategies. Many books have been written to illustrate the characteristics of option strategies (Bookstaber and Clarke, 1983). Option is helpful for investors to control the different strategies but facing the option in the market, designing the right and choosing the right strategy is still a big problem. In order to address this issue, this chapter builds a model and uses the quantitative analysis method to solve the problems of designing option strategies empirically.

This chapter uses the option of HSI as an example, so the foundation of building the model is good prediction of HSI. The predicting data in this chapter is the result of a distortion of Monte Carlo simulation which has been proved accurate in most cases. In fact, all financial companies have their own ways of predicting the market and they trust the prediction to a certain degree. The main objective of this chapter is to build the model to solve the problem of designing option strategies empirically. So it is assumed that the market trend predicted in this chapter is accurate.

The model in this chapter is built to solve the problems of designing option strategies empirically. Some standard measurements are used to build the model such as VaR, CVaR, expected return, return rate and so on. There are, usually, scores of options to choose from at any given time. What’s more, the complex strategies which combine different options can result in a number of different strategies running into thousands. The model in this chapter can rank the different strategies and provide different efficient frontiers by the measures mentioned above. Different evaluation criteria are calculated as references for investors of different risk appetites. It is easy for investors to estimate potential risks and benefits to optimize their investment strategies.

This chapter is composed of four main parts. The first part is the introduction which outlines the background, describes previous research works in this field and illustrates the basic concepts and the significance of building the proposed model. The second part is the theoretical background and the model. In this part, the theory and the process of building the model are described. The third part is the result of the model. In this part, the result is discussed and the model is demonstrated to be useful in reality, by using real cases. The last part is the conclusion which summarizes the significance of this chapter and the meaning of the model proposed in this chapter.

5.2 Theoretical Background and Model

The model proposed in this chapter is based on the HSI (Heng Seng Index), implying that the option strategies yielding profit depends on the trend of HSI. The option is European style option which has an expiration date, T, and it can only be executed at the expiration date but it can be traded at the market as a kind of merchandise. It has been mentioned that investors who use option strategies for arbitrage usually try to find the option which strays away from its normal value and earn the difference in value by buying and selling the option. There are dozens of option strategies which can be used for arbitrage and it is impossible to list all option strategies in one paper. However, according to the position of option, it can be divided into four kinds of basic styles and composite types. In this chapter, four kinds of basic styles are discussed and one kind of composite type is chosen as an example to build the model.

5.2.1 The Model of Basic Style

In the following, the model of strategy short call is built. Short call means selling a contract and the buyer of the contract has the option to buy the underlying asset at the agreed price at the expiration date. The payoff of this kind of strategies can be expressed in piecewise functions. The payoff of this strategy can be defined as:

$$Payoff = \begin{cases} n \times p_c^t & S_T \le K_c \\ n \times p_c^t - n \times (S_T \le K_c) & S_T > K_c \end{cases} \tag{5.1}$$

In this equation, $Payoff$ is the profit derived by selling a call option. p_c^t is the price of each call option at time t. K_c is the strike price of this call option. S_T is the price of the underlying asset at the time of expiration. The first part is that the contract becomes out-of-the-money while the other part is at-the-money. For the sake of simplicity, the parameter n is assumed to be "1" in the following part of this chapter.

In fact, investors who use option for arbitrage usually use short-swing trading. That means the strategy is usually stopped before expiration. In this condition, the payoff can be defined as:

$$Payoff = p_c^t - p_c^{t'} \tag{5.2}$$

where parameter $p_c^{t'}$ means the value of the call at time t' and $t < t' < T$. It is impossible to predict values of $p_c^{t'}$. It has been mentioned above that B-S model is popular for calculating the price of options. So $p_c^{t'}$ can be obtained from the function:

$$p_c^{t'} = blsprice(S_{t'}, K_c, r, T - t', v, y) \tag{5.3}$$

In Equation (5.3), r is annualized, continuously compounded risk-free interest rate of return and v is annualized asset price volatility. y is annualized, continuously compounded yield of the underlying asset and in This chapter it is assumed to be zero. So in Equation (5.3), once $S_{t'}$ and v are estimated, value of $p_c^{t'}$ can be calculated.

In this chapter, $S_{t'}$ and v can be predicted by using one kind of Monte Carlo simulation. The result of Monte Carlo simulation is a distribution of $S_{t'}$. The expected payoff of this strategy is expressed as:

$$E[payoff] = \sum_{i=1}^{n}\left(p_c^t - p_c^{it'}\right)P\left(S_{t'}^i\right) \tag{5.4}$$

$P\left(S_{t'}^i\right)$ means the probability of value of the underlying asset $S_{t'}^i$ at time t' being $P\left(S_{t'}^i\right)$. The expected payoff is one of the evaluation criteria. In Equation (5.4), parameter $p_c^{it'}$ can be replaced by Equation (5.3). The expected return is only dependent on distribution of the underlying asset. In the following part, VaR, CVaR and return rate are calculated to meet different risk appetites of different investors.

The definition of value-at-risk is a threshold value that defines the maximum mark-to-market loss on the portfolio over the given time horizon. In this problem, it can be expressed as:

$$VaR_\alpha(payoff) = -\inf\left\{l \in R : F_{payoff}(l) \geq \alpha\right\} = p_c^t - p_c^{jt'} \quad j = n\alpha \tag{5.5}$$

where j is the jth $p_c^{jt'}$ after ranking the $p_c^{jt'}$ in accordance with order of return from bad to good. Using the same method, CvaR can be expressed as:

$$CVaR_\alpha(payoff) = \frac{1}{\alpha}\left[\sum_{i}^{j}\left(p_c^t - p_c^{it'}\right)P\left(S_{t'}^i\right)\right] \quad j = n\alpha \tag{5.6}$$

All parameters in Equation (5.6) have the same assumptions and meanings as the parameters in Equation (5.5). The last evaluation criterion which needs to be calculated in this model is the return ratio. In the models in this chapter, the methods to calculate this parameter are different for different strategies.

For strategies of selling the option, there is no the definition of input-output ratio, but there is ratio of potential income and risk, so the ratio can be viewed as the quotient of expected return divided by the potential loss.

$$R = \frac{E[payoff]}{\left|\frac{1}{\sum_{i}^{k}P\left(S_{t'}^i\right)}\left[\sum_{i}^{k}\left(p_c^t - p_c^{it'}\right)P\left(S_{t'}^i\right)\right]\right|} \tag{5.7}$$

where k is the number which, if $i \leq k$ the return corresponding to $S_{t'}^i$ is negative otherwise the return is positive.

As to the strategies of buying option, it can be defined as:

$$R = \frac{E[payoff]}{C} \tag{5.8}$$

where C is the cost of buying the option.

5.2.2 The Model of Composite Style

The model of composite style can be viewed as a combination of different basic styles. In this chapter, the composite style of bull spread is used as an example. In option trading, a bull spread is a bullish, vertical spread options strategy that is designed to profit from a moderate rise in the price of the underlying security. Because of put-call parity, a bull spread can be constructed using either put options or call options. If constructed by using calls, it is a bull call spread, otherwise it is a bull put spread. The bull call spread is constructed by buying a call option with a low exercise price, K_{c1}, and selling another call option with a higher exercise price, K_{c2}. They usually have the same expiration date. There are dozens of call options that can be chosen to construct this kind of strategies. The number of this kind of strategies can be very large and it is impossible for investors to analyze and find the optimal strategy for arbitrage. It is the simplest strategy in composite style. As to strategies such as butterfly and condor, constructed with four or more basic styles, the number is sizeable. They are more difficult to analyze. So the evaluation criteria are necessary for investors to find optimal strategy for arbitrage.

In the following part the bull call spread is used as an example of composite style. The model of the bull call spread strategy is derived from the model of short call to calculate the evaluation criteria of this strategy. As to the other composite, it can be derived from the model of short call, the same as this strategy. The notation system and assumptions are the same as in the model above. And the lower exercise price call option is marked with subscript “1”, while the other is marked “2”. The payoff can be expressed as:

$$payoff = \begin{cases} p_{c2}^{t} \times n_{c2} - p_{c1}^{t} \times n_{c1} & S_t \le K_{c1} \\ (S_t - K_{c1}) \times n_{c1} + p_{c2}^{t} \times n_{c2} - p_{c1}^{t} \times n_{c1} & K_{c1} < S_t \le K_{c2} \\ (K_{c2} - K_{c1}) \times n_{c2} + p_{c2}^{t} \times n_{c2} - p_{c1}^{t} \times n_{c1} & K_{c2} < S_t \end{cases} \tag{5.9}$$

And it is the same as the model above. In reality it is usually short-swing trading for arbitrage and for the sake of simplicity, assume $n_{c1} = n_{c2} = 1$. Then Equation (5.2) for this model is:

$$payoff = \left(p_{c1}^{t'} - p_{c1}^{t}\right) + \left(p_{c2}^{t} - p_{c2}^{t'}\right) \tag{5.10}$$

In Equation (5.10) parameters of $p_{c1}^{t'}$ and $p_{c2}^{t'}$ can also be calculated by using the B-S model, described in Equation (5.3). And the expected payoff of this strategy is as follows.

$$E\left[payoff\right] = \sum_{i=1}^{n} \left\{ \left[\left(p_{c1}^{it'} - p_{c1}^{t}\right) + \left(p_{c2}^{t} - p_{c}^{it'}\right) \right] P(S_{t'}^{i}) \right\} \tag{5.11}$$

It has been mentioned above that distribution of the underlying asset $S_{t'}$ and the parameters in B-S model can be obtained by using Monte Carlo simulation. Then the expected return can be calculated by substituting Equation (5.3) into the equation above. Of course, the evaluation criteria of VaR, CVaR and return ratio are also needed to be calculated. The model to calculate these criteria can be derived from the model above.

The mathematical equation of VaR of this strategy is:

$$\begin{aligned} VaR_{\alpha}(payoff) &= -\inf\left\{l \in R : F_{payoff}(l) \geq \alpha\right\} \\ &= \left(p_{c1}^{jt'} - p_{c1}^{t}\right) + \left(p_{c2}^{t} - p_{c2}^{jt'}\right) \qquad j = n\alpha \end{aligned} \tag{5.12}$$

The parameters in this equation have the same meanings as in Equation (5.5).

The mathematical equation of CvaR of this strategy is:

$$CVaR_{\alpha}\left(payoff\right) = \frac{1}{\alpha}\left[\sum_{i=1}^{j}\left(\left(p_{c1}^{jt'} - p_{c1}^{t}\right) + \left(p_{c2}^{t} - p_{c2}^{jt'}\right)\right)P(S_{t'}^{i})\right] \qquad j = n\alpha \tag{5.13}$$

The definition and assumptions of VaR and CvaR are the same as the model in the basic style part but the method of calculating the return ratio is different from the former. The return ratio of this strategy can be calculated by using Equation (5.9). This strategy is constructed by buying a lower exercise price call option and selling a higher exercise price call option and the two options have the same expiration date and underlying asset. In these two kinds of options, the lower exercise always has the higher price. So constructing this kind of strategy always leads to a cost. So the return ratio belongs to condition that the input-output ratio can be calculated by using Equation (5.9) described when building the model of return ratio in the basic style part. This can be written as:

$$R = \frac{E\left[payoff\right]}{p_{c1}^{t} - p_{c2}^{t}} \tag{5.14}$$

The expected payoff is calculated in Equation (5.11) and the denominator is the cost incurred for constructing this strategy.

In this part, two models are built to support the investment for arbitrage by trading options. The first is the model for the basic style while the other is for the composite style, constructed by a different option. For other strategies, the models can be built in the same way. There are many options trading in the market at any given time, and the number of different combinations of these strategies is huge. It is impossible to find a suitable strategy for arbitrage. In the above model, four evaluation criteria are proposed for optimization of the strategy automatically and to meet different risk aptitudes by ranking the strategies based on different criteria. In the following part, a real case is used as an example to prove the validity of this method.

5.3 Results

In this part, a real case is used to prove the validity of the model. We made the

Mote Carlo simulation on 28 September 2011 to predict the trend of HSI. In fact, distribution of the HSI can be obtained by this simulation. The means are described in the following figure. Fig. 5-1 presents the trend of HSI as predicted by Monte Carlo simulation.

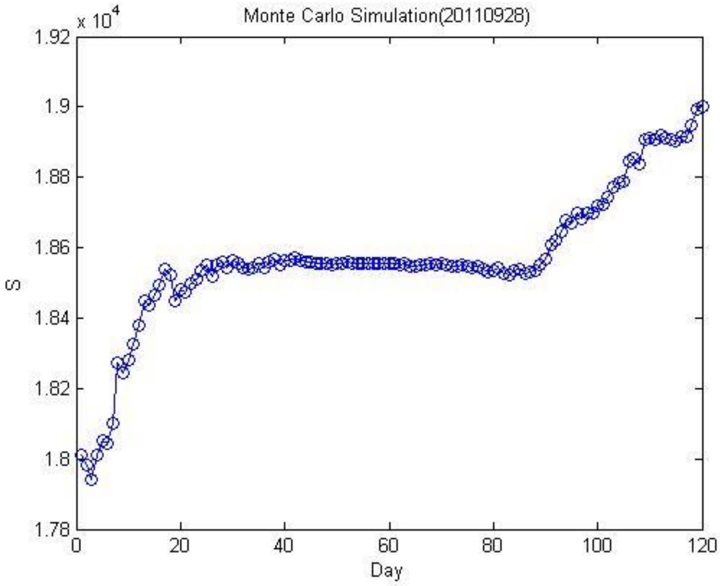

Fig. 5-1 Result of Monte Carlo simulation.

The conclusion that can be drawn from the figure is that the Heng Seng Index will rise in the next 120 days. But it will drop in the next few days. So it can be derived that for the short-term investment in options for arbitrage, selling a call or buying a put will yield a profit. In the former part, four evaluation criteria are pointed out and the model is built to calculate these four criteria to support investors in identification of the optimal strategy which can meet their taste. Here, we use one of the four criteria, VaR, as an example to rank the basic option strategies as a reference for investors.

Table. 5-1 Ranking of basic style by VaR.

	Expiration date	code name	VaR	SharpeRatio
Short call	11/2011	C18800	813.237	0.811
	11/2011	C19000	807.176	0.883
	11/2011	C20000	802.545	0.968
Long call	12/2011	C18400	-10.000	0.475
	12/2011	C20200	-14.000	0.478
	12/2011	C19800	-19.999	0.474
Short put	12/2011	P14600	-49.000	0.950
	11/2011	P17400	-68.000	0.876
	12/2011	P17800	-100.000	0.837
Long put	12/2011	P16400	899.975	0.934
	11/2011	P15800	897.999	0.997
	12/2011	P8800	896.626	1.000

Table. 5-1 above lists the results of ranking of the four basic styles by the order of Var. Some basic information is also listed in the table. Based on this the optimal strategy for arbitrage is selling the call option C18800 or buying the put option P16400. By observing the real date of option as on 28 September 2011 and 29 September 2011, the result of using these two option strategies for arbitrage is as listed in the table below.

Table. 5-2 The Result of Using Option Strategies for Arbitrage.

Contract	28/09/2011	30/09/2011	Payoff	Return rate
Short HSI 11/2011 C18000	639	470	165	25.9%
Long HSI 11/2011 P16400	586	687	101	17.2%

Table. 5-2 shown above proves that these two strategies are wonderful for arbitrage and prove that the model is useful to support the investment for using options for arbitrage. This can work well if the trend of the underlying asset can be predicted accurately.

The results illustrated above are just for the four basic style option strategies of arbitrage and only results ranked by VaR are listed. In fact, the model can work for all option strategies such as bull spread, butterfly, condor and so on and all of evaluation criteria can be calculated easily. They are useful criteria for the investors to design effective option strategies for arbitrage. It can support the investor to find the optimal option strategy.

5.4 Conclusions

This chapter is the first to propose automatic designing of option strategies, using evaluation criteria, to support investors in finding the optimal strategy in the area of using options for arbitrage. Option has been widely viewed as a powerful tool in both risk management and arbitrage but the complexity of its usage always keeps investors away. This is a big stumbling block in the development of the option as a financial instrument. Though considerable research has been conducted to introduce the method of using option strategy, few works have used standard criteria to evaluate the effects of different strategies and there is no work which can support the investor in design of the optimal strategy automatically. Traditional investors usually design their strategies empirically, which brings in a lot of problems. This chapter builds models (illustrated in the second part) to solve this problem. The model is introduced by dividing it into two parts. The first part is the model for the basic styles while the second part is the model for the composite styles. The models are proved valid by applying the model to real data and the results are shown in the third part of this chapter.

Though this chapter makes great contribution to both evaluation and optimization of option strategies used for arbitrage, the information the model can provide is not enough. In future work, information of the parameters of Greeks in option can be provided, which will make the model integrated and more effective.

Chapter 6 A Novel Mean Reversion-based Local Volatility Model

Local volatility is deterministic volatility, which is a function of underlying asset price and time. If the local volatility and time are given, the underlying asset price can be calculated from this function. This chapter proposes a novel local volatility model with mean-reversion process and overcomes the weakness of traditional local volatility. After the local volatility is precisely modeled, a surface fitting scheme is used to recover the local volatility surface.

6.1 Introduction

The mean-reversion phenomenon of stock prices was firstly proposed by Keynes (1936) that 'all sorts of considerations enter into market valuation which are in no way relevant to the prospective yield' (p.152). When the stock prices diverge from its long-run mean level, they will eventually return back. The larger the deviation is, the higher probability stock prices will return with. In the empirical study of Cecchetti et al. (1990), they found that the asset price usually vibrated within 60 percent confidence interval of its long-run mean level and the asset price would finally return to its mean level.

Hence, the mean-reversion process is common in stock market. Bessembinder et al. (1995) studied the term structure of commodity futures prices and obtained a conclusion that mean reversion exists in equilibrium asset price. Moreover, Engle and Patton (2001) found that the mean-reversion phenomenon also existed in volatility of asset price when they sought volatility model to predict volatility. That is a high volatility of underlying asset price would eventually evolve to its long run mean level. In this chapter, we focus on the evolution of local volatility of underlying asset price and propose a novel local volatility model with mean-reversion process.

Dupire (1994) presented a deterministic equation to calculate the local volatility from option price basing on the assumption that all call options with different strikes and maturities should be priced in a consistent manner. Sepp (2002) used market implied volatility to extend the Dupire's Local Volatility Formula. However, from our empirical analysis, we find that both of these models are unstable. The local volatility calculated from these models may reach 800 percent or above at some cases. This is irrational. To improve these weaknesses, we propose a novel robust local volatility model with mean-reversion process. Upon the traditional local volatility model, we design a mean reversion term and plug into traditional formula. With this term, when the computed local volatility is too high or too low, our model can revert it and make local volatility return to its long-run mean level.

6.2 Motivations

Local volatility is the deterministic function of the underlying asset price and time. This means that the local volatility has the ability of forecasting future stock price trend. Most of current literatures focus on improving the computation preciseness of local volatility. Coleman et al. (1999) used spline to present local volatility function. They obtained this function by solving a nonlinear optimization problem to closely fit these option prices. Crépey (2003) found that local volatility obtained from real option prices was usually unstable. He proposed to use Tikhonov regularization method to control local volatility and minimized the distance between calibration price and market price to get optimal Tikhonov regularization parameters.

However, there are still little literatures applying mean reversion to improve the local volatility model. Black (1990) proposed that a volatility model with mean reversion could help to interpret why volatility and risk premium declined when wealth rose. Fouque et al. (2000) found that the stochastic volatility reverted slowly to its mean level but fast when the maturity was large. Feng et al. (2010) analyzed Heston stochastic volatility model and found that the maturity was large when compared to rate of mean-reversion. In this chapter, we propose that the local volatility model also own the property of mean reversion. The larger local volatility deviates from its mean level, the greater rate the local volatility will revert with.

After the local volatility is precisely modeled, we use Monte Carlo simulation to estimate future stock trend. We use the normalized distribution with mean of the risk-free interest rate and variance of the local volatility. Finally, the path-dependent trend of stock price is simulated from local volatility.

6.3 Mean reversion-based local volatility model

6.3.1 Local Volatility Model

Local volatility is the deterministic function of the underlying asset price and time. Dupire (1994) proposed the local volatility model formula as follows.

$$\sigma_{loc}^2\left(K,T\right)=2\frac{\dfrac{\partial C_{market}}{\partial T}+\left(r\left(T\right)-d\left(T\right)\right)K\dfrac{\partial C_{market}}{\partial K}+d\left(T\right)C_{market}}{K^2\dfrac{\partial^2 C_{market}}{\partial K^2}} \tag{6.1}$$

Sepp (2002) uses market implied volatility to express the Dupire Local Volatility

Formula as follows.

$$\sigma_{loc}^2\left(K,T\right)=2\frac{\dfrac{\sigma_{imp}}{T-t}+2\dfrac{\partial\sigma_{imp}}{\partial T}+2\left(r\left(T\right)-d\left(T\right)\right)K\dfrac{\partial\sigma_{imp}}{\partial K}}{K^2\left(\dfrac{\partial^2\sigma_{imp}}{\partial K^2}-d_1\sqrt{T-t}\left(\dfrac{\partial\sigma_{imp}}{\partial K}\right)^2+\dfrac{1}{\sigma_{imp}}\left(\dfrac{1}{K\sqrt{T-t}}+d_1\dfrac{\partial\sigma_{imp}}{\partial K}\right)^2\right)} \tag{6.2}$$

where

$$d_1=\frac{\ln\left(S/K\right)+\left(r\left(T\right)-d\left(T\right)+\frac{1}{2}\sigma_{imp}^2\right)\left(T-t\right)}{\sigma_{imp}\sqrt{T-t}}$$

However, in the financial practice, we find that both of these equations are not stable, especially when we use the long term expiry of options from HSI. The local volatility will fluctuate greatly and suffer a lot from the errors caused by implied volatility calculated from long term options. As is shown in Fig. 6-1, the local volatility has reached 800 percent where using long term maturity option and large underlying asset price. Besides, there also exist many negative values of local volatility calculated from these two formulas.

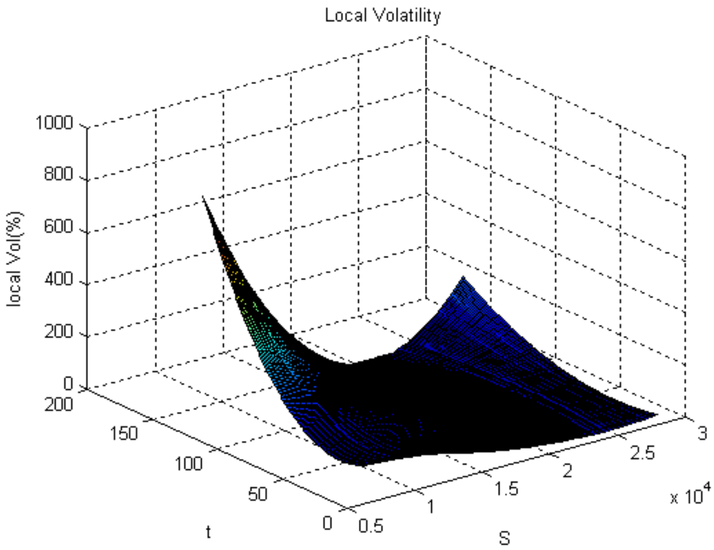

Fig. 6-1 Local Volatility Surface.

Therefore, to fix this problem, we propose to add a mean-reversion term to the Sepp's Equation when deriving local volatility from implied volatility. This mean-reversion term will help to correct the errors caused by computation. To make our local volatility more robust, we consider the computation of local volatility with the existence of market shocks or jumps which ensure the local volatility model more suitable to the realized market.

6.3.2 Mean-reversion Process

As local volatility is a function of time and underlying asset price, we fix the underlying asset price[◊] and analyze the mean-reversion process of local volatility against time. Implied volatility is a function of maturity and strike price. However, the series of maturities of different options is discrete. In order to analyze the relationship between underlying asset price and implied volatility, Sepp (2002) proposed a local volatility calculated from implied volatility based on Dupire's works. In his model, the local volatility is a function of continuous time. Hence, to improve this model, we propose a mean-reversion term and apply this term to local volatility model.

Assume that X_t is the long-run mean level of local volatility, θ is the reverting rate. The further local volatility departs from its mean level, the greater the reverting rate will be. Based on this assumption, we formulate our novel local volatility with mean reversion as follows.

$$Y_t' = X_t + \left(Y_t - X_t\right)e^{-\theta|Y_t - X_t|} \tag{6.3}$$

where $X_t = \frac{1}{t}\sum_{i=1}^{t}\sigma_{loc}\left(S,i\right)$ and $\theta \geq 1$. θ can be determined from real market data. Y_t is the local volatility calculated by Sepp's formula. Y_t' is the volatility computed by our novel local volatility model. We view the distance between local volatility and its mean level as departure degree. If this distance increases dramatically, the right hand side of (6.3) will also decrease deeply. Hence, we assert that if the observed horizon is long enough, Y_t' will finally converge to its long-run mean level.

Hypothesis 1: If a local volatility is given with a mean-reversion process as follows,

$$Y_t' = X_t + \left(Y_t - X_t\right)e^{-\theta|Y_t - X_t|}$$

Then, Y_t' *will finally converge to its long-run mean level.*

Proof:

$$E\left[Y_t'\right] = E\left[X_t\right] + E\left[\left(Y_t - X_t\right)e^{-\theta|Y_t - X_t|}\right]$$

[◊] Since the local volatility is a function of underlying asset price and time and the time is assumed to be fixed, we will go through the proof of Y_S' with different prices S converging to its long-run mean level. This is the same process as underlying asset price is fixed mentioned above. Therefore, we only discuss one case in this paper.

$$= E\left[X_t\right] + E\left[Z_t e^{-\theta|Z_t|}\right]$$

$\left(where,\ e_t = Z_t - Y\right)$

$$= E\left[X_t\right] + \int_{-\infty}^{+\infty} Z_t^2 e^{-\theta|Z_t|} dZ_t$$

$$= E\left[X_t\right] + \int_{0}^{+\infty} Z_t^2 e^{-\theta Z_t} dZ_t + \int_{-\infty}^{0} Z_t^2 e^{\theta Z_t} dZ_t$$

$\otimes$

In $\otimes$, we calculate two definite integrals respectively as follows.

$$\int_{0}^{+\infty} Z_t^2 e^{-\theta Z_t} dZ_t = e^{-\theta Z_t} \int_{0}^{+\infty} Z_t^2 dZ_t + Z_t^2 \int_{0}^{+\infty} e^{-\theta Z_t} dZ_t = \frac{1}{3} Z_t^3 e^{-\theta Z_t} \Big|_0^{+\infty} - \frac{1}{\theta} Z_t^2 e^{-\theta Z_t} \Big|_0^{+\infty}$$

$$= \lim_{Z_t \to +\infty} \frac{1}{3} Z_t^3 e^{-\theta Z_t} - 0 - \left(\lim_{Z_t \to +\infty} \frac{1}{\theta} Z_t^2 e^{-\theta Z_t} - 0 \right) = 0$$

And,

$$\int_{-\infty}^{0} Z_t^2 e^{\theta Z_t} dZ_t = e^{\theta Z_t} \int_{-\infty}^{0} Z_t^2 dZ_t + Z_t^2 \int_{-\infty}^{0} e^{\theta Z_t} dZ_t = \frac{1}{3} Z_t^3 e^{\theta Z_t} \Big|_{-\infty}^{0} + \frac{1}{\theta} Z_t^2 e^{\theta Z_t} \Big|_{-\infty}^{0}$$

$$= 0 - \lim_{Z_t \to -\infty} \frac{1}{3} Z_t^3 e^{\theta Z_t} + 0 - \lim_{Z_t \to -\infty} \frac{1}{\theta} Z_t^2 e^{\theta Z_t} = 0$$

Then, $\otimes$ can be modified as follows.

$$\therefore E\left[Y_t^{'}\right] = E\left[X_t\right]$$

This means that for a long horizon of observations, the expected new local volatility will equal to that of the long-run mean level. Hence, the new local volatility will finally revert to its long-run mean level.

□

Therefore, combining equation (6.2) and (6.3), we propose our novel mean-reversion local volatility model as follows.

$$\sigma_{loc}^{'}\left(S,t\right)\Big|_{S=S_0} = \frac{1}{t}\sum_{i=1}^{t}\sigma_{loc}\left(S,i\right) + \left(\sigma_{loc}\left(S,t\right) - \frac{1}{t}\sum_{i=1}^{t}\sigma_{loc}\left(S,i\right)\right) e^{-\theta\left|\sigma_{loc}(S,t) - \frac{1}{t}\sum_{i=1}^{t}\sigma_{loc}(S,i)\right|}\Big|_{S=S_0} \tag{6.4}$$

where S_0 is a given underlying asset price.

After the local volatility is precisely modeled, we use Monte Carlo simulation to estimate future underlying asset price trend. We apply the normal distribution with mean of the risk-free interest rate and variance of the local volatility. Finally, the path-dependent trend of underlying asset price is simulated from local volatility.

6.4 Local Volatility Surface

Typically, Dupire (1994) presented a deterministic equation to calculate the local volatility from option price based on the assumption that all call options with different strikes and maturities should be priced in a consistent manner.

Nevertheless, this deterministic function also suffers two weak points. First, the shortcoming is intrinsic problem of the Dupire's equation. The indeterminacy of the equation may cause the local volatility on the left to be extremely large or tremendous small. Second, because local volatility is a function of both strike and time to maturity and it is possible that not all the same list of strikes are available at each time to maturity, the number of local volatility is finite, usually is not enough for further calculation and applications. As a result, researchers are inclined to use interpolation method to obtain a series data of local volatility for further calculation.

To fix the first problem, we propose a novel local volatility model with mean-reversion process described in the previous section. In this section, we focus on the second problem by using Least Squares Method. We define a surface function $\varphi(x)$ as follows. The left hand side of the function represents the fitted local volatility.

$$\varphi(S,t) = \sum_{i=0}^{M-1}\sum_{j=0}^{T-1} A_{ij} S^i t^j \tag{6.5}$$

where *A* is a *MxT* matrix. *S* is the underlying asset price. *t* is time, measured in year. Our aim is to find an optimal parameter matrix *A* such that the difference between fitted local volatility and real local volatility is minimal. The errors function is defined as follows.

$$E_A = \sum_{k=1}^{N} \omega(S_i, t_j) \left[\sum_{i=0}^{M-1}\sum_{j=0}^{T-1} A_{ij} S_i t_j - \sigma(S_i, t_j) \right]^2 \tag{6.6}$$

where *k=1,…,N*, *N* means the total number of local volatilities, $\sigma(S_i, t_j)$ is the real local volatility with underlying asset price S_i and time t_j, $\omega(S_i, t_j)$ is the weighted matrix at different points. Obviously, if we minimize the summation of errors, we will obtain an optimal parameter matrix for the surface function. Since a parabolic function can reach an optimal value at a given variable range, we differentiate (6.6) and get a partial differential equation of summation error function to a given parameter.

$$\frac{\partial E_A}{\partial A_{ij}} = 0 \tag{6.7}$$

We let the partial differential result to be zero. In this way, the optimal value of the parabolic function is derived. After moving the real local volatility to right hand side, (6.7) is transformed into (6.8).

$$\begin{pmatrix} (\varphi_{00},\varphi_{00})(\varphi_{00},\varphi_{01}) & \cdots & (\varphi_{00},\varphi_{M-1T-1}) \\ \vdots & \ddots & \vdots \\ (\varphi_{M-1T-1},\varphi_{00})(\varphi_{MT},\varphi_{12}) & \cdots & (\varphi_{M-1T-1},\varphi_{M-1T-1}) \end{pmatrix} \begin{pmatrix} A_{00} \\ \vdots \\ A_{M-1T-1} \end{pmatrix} = \begin{pmatrix} (\varphi_{00},\sigma) \\ \vdots \\ (\varphi_{M-1T-1},\sigma) \end{pmatrix} \tag{6.8}$$

where $\varphi_{00} = S^0 t^0$, $\varphi_{01} = S^0 t^1$,…., $\varphi_{M-1T-1} = S^{M-1} t^{T-1}$. And $(\varphi_{00},\varphi_{00}) = \sum_{k=1}^{N} S_k^0 t_k^0 \square S_k^0 t_k^0$, $(\varphi_{M-1T-1},\varphi_{M-1T-1}) = \sum_{k=1}^{N} S_k^{M-1} t_k^{T-1} \square S_k^{M-1} t_k^{T-1}$, $(\varphi_{M-1T-1},\sigma) = \sum_{k=1}^{N} S_k^{M-1} t_k^{T-1} \square \sigma_k$. The optimal parameter matrix A can be obtained by solving (6.8).

6.5 Empirical Tests

Option price is the price of a contract which specifies a price that an underlying asset will be sold at in a future given time. It is a difference between current underlying asset price and future price at a given time. This means that this option price contains some views of investors or market participants that whether future market goes up or down. In this chapter, we adopt the daily closed option prices of Hang Seng Index (HSI), which are provided by Hong Kong Exchanges and Clearing Limited (HKECL). Since the daily closed option price contains market information about next day, next week, or even next month, this price also contains the power of predicting the future trend of underlying asset price.

However, the data provided by HKECL usually contains noise data, which is useless for empirical study. Some data entries contains bid, ask, and closed price of options. But there is no trading volume. These prices do not reflect market views and should be considered as noise data. Besides, when the trading day is closed to end of month, the price of near expiry term options will become abnormal. The local volatilities calculated from these option prices are also abnormal. To overcome this problem, we introduce an expiry rolling process to compute local volatility. Options are rolled to next expired month starting from the 5th trading day prior to their current expired month. Moreover, when using option market data to calculate implied volatility, there is a rolling process for the options which will be expired. The rolling process is that the time to maturities of current expiry month options with less than 5 trading days are calculated by using the expiry date of next expiry month contract options as the expired date of these options.

As mentioned above, the Monte Carlo method is adopted to simulate underlying asset price with local volatility as variation. The analytic solution of stochastic differential equation of Geometric Brownian motion shown as follows is used to calculate future prices.

$$S_{t+1} = S_t \exp\left(\left(\mu_t - \frac{V_t}{2}\right)t + \sqrt{V_t}W_t^S\right)$$

$$W_t^S = \sqrt{dt}\varepsilon$$

where S_t is the underlying asset price at time t. For a given price S_t, V_t is the corresponding local volatility at time t. μ_t is the drift rate, a function of time t. In This chapter, we use the HIBOR[§] as the drift rate. W_t^S is a Wiener process, which is generated by normal distribution with mean of 0 and variance of 1. ε is a random term following a standard normal distribution.

To compare the performance of mean-reversion local volatility model with traditional local volatility model, we use the forecasting power as criterion. We compare the prediction power from three aspects, one-day-ahead prediction (ODA), five-day-ahead prediction (FDA), and price position prediction (PPP) precision. For position prediction precision, RMSE is applied to measure average errors.

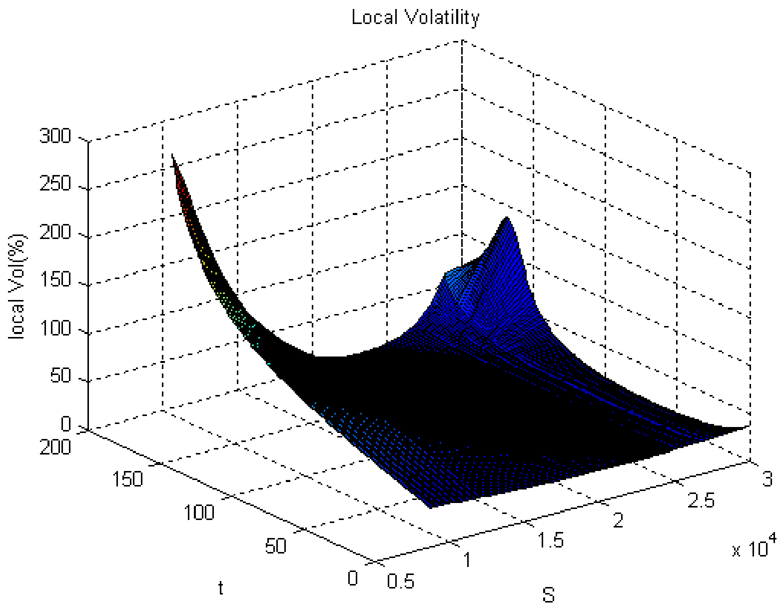

Fig. 6-2 Volatility Surface by Traditional Local Volatility Model.

[§]HIBOR is Hong Kong Interbank Offered Rate, the annualized offer rate that banks in Hong Kong offer for a specified period ranging from overnight to one year, normally including overnight, 1 week, 2 weeks, 1 month, 2 months, 3 months, 6 months, and 1 year.

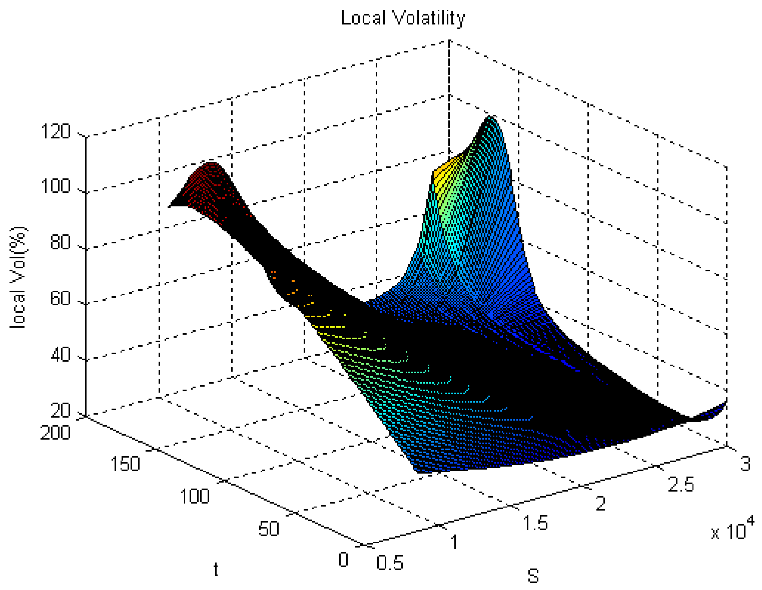

Fig. 6-3 Volatility Surface by Mean-reversion Local Volatility Model.

Fig. 6-3 shows that our novel can efficiently control local volatility within a rational range while the local volatility in Fig. 6-2 varies greatly and departs far from its mean level. When comparing the forecasting power, we use the market option data from HKECL with a horizon of 17 months, from Jan 2011 to May 2012.

Table. 6-1 Comparison of Forecasting Power.

Model	ODA	FDA	PPP
CEV Model	50.92%	49.75%	259.87
Traditional Local Volatility Model	52.69%	51.46%	33.16
Mean-reversion Local Volatility Model	57.79%	51.73%	65.01

While comparing the position prediction precision, we use the mean absolute error as comparison criteria. For both one-day-ahead prediction and five-day-ahead prediction shown in Table. 6-1, our mean-reversion local volatility model has a better performance than the other two models. There are 57.79% and 51.73%, respectively. This means that as our model can control the local volatility varies within a rational range, this is little fluctuation after Monte Carlo simulation. The prediction of our model can be more stable than that of the traditional local volatility model. Besides, since we have constructed the three-dimension surface for both mean-reversion local volatility model and traditional local volatility model, the local volatilities are different at different days for simulation. However, CEV is a constant elasticity volatility model, which assuming the volatility of volatility is constant. This is unsuitable for real market. Hence, from the results of Table. 6-1, we can find that the prediction power is poor than mean-reversion local volatility model.

6.6 Conclusions

From literature review, it is found that there is little works covering application of mean-reversion process to local volatility surface. This chapter proposes a novel mean-reversion local volatility model based on empirical study of Hong Kong options market. The local volatility calculated from traditional local volatility model is adjusted by a mean-reversion term. If the local volatility is far from its mean level, this term will reverse it. The further it departs from mean level, the greater rate it will be reverted. After that, least squares method is used as surface fitting scheme to recover local volatility surface. This surface is then used for Monte Carlo simulation. For a given price, local volatility can be abstracted from the surface at different simulated time. Meanwhile, in the empirical tests section, the option price is processed by a rolling procedure. The expiries of options with a few trading days to expiry are rolled into next expiry so that the local volatility computed by these options can be useful. Finally, our empirical tests results show that the prediction power of our mean-reversion local volatility model is better than that of traditional local volatility model and CEV model.

Chapter 7 Regression-based Correlation Modeling for Heston Model

In chapter 3, the correlation of Heston model is considered to be dynamic rather than constant assumed by traditional Heston model. This chapter further explores this correlation and proposes a regression-based dynamic correlation Heston model. The correlation is estimated by three different regression models between the volatility and underlying asset price. They are respectively simple regression model, polynomial regression model, and auto-regression model. The prediction performance is compared among these models in the empirical tests.

7.1 Introduction

Heston model is a stochastic model. It assumes the underlying asset price and volatility of prices follow Geometric Brownian motions. These two stochastic processes have a constant correlation. To further study Heston model, Jose et al. (2008), Christoffersen et al. (2006) and Sylvia and Leopold (2008) proposed to model a variance process of Heston model as a multifactor Wishart process and applied Fast Fourier Transform to price options. Eric et al. (2010) derived an accurate analytic formula for time-dependent Heston model to price vanilla options by extending the parameter of volatility of volatility. Besides, Sepp (2008) applied realized variance with Heston model to price options and considered jumps of asset returns and variance. Though many other studies have focused on the numerical simulation of Heston model, there is little literature studying the correlation coefficient of Heston model.

Saikat (1998), Josep et al. (2004), and Kahl and Jackel (2006) studied the correlation of Heston model and analyzed correlation of Heston model as constant. Hobson (2004), Asaia and McAleer (2009) considered the correlation coefficient as dynamic parameter of Heston model. Asaia and McAleer (2009) used Markov Chain Monte Carlo procedure to estimate the dynamic correlation of Heston model. However, all of their showed great inconvenience and complications during real market operation.

Hence, to overcome these weaknesses, we proposed a regression-based dynamic correlation Heston model. Our contributions contain two aspects. First, we consider the correlation of stochastic model as dynamic rather than constant. From the literature review, we find that most of current studies are still focus on constant correlation of stochastic models. Second, we propose to use multivariate regression model to estimate dynamic correlation of Heston model. Although some recent papers covered dynamic correlation, they were complicated and were not suitable for real time calculation, especially for high frequency trading. However, our model can be computed quickly and efficiently.

7.2 Heston Model

The Heston model is more complicated than the CEV model. It takes the mean-reversion term into consideration and the assumption is that the two processes of underlying asset price and volatility are stochastic processes with a constant correlation to each other. The original Heston model is defined as follows.

$$dS_t = \mu(t)S_t dt + \sqrt{V_t} S_t dW_1$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma\sqrt{V_t} dW_2 \tag{7.1}$$

where θ is the long-run mean level of volatility, κ is the speed of instant volatility returning to long-run mean level, σ is volatility of volatility. These three parameters satisfy the condition of $2\kappa_t\theta > \sigma^2$ and ensure the process of V_t to be strictly positive. Besides, W_1 and W_2 are two standard Wiener process and have a correlation of ρ.

We find that the constant assumption does not hold in HSI market. Fig. 7-1 shows comparison of VHSI and HSI from June 2008 to July 2011. The red line represents the time series data of HSI. The blue line represents the time series of VHSI. The two are obviously highly correlated. When HSI went down during the period between June 2008 and November 2008, the trend of VHSI was obviously upward. When HSI climbed from March 2009 to Nov 2009, VHSI also declined gradually. When the price fluctuated from January 2010 to July 2011, VHSI also experienced high fluctuations.

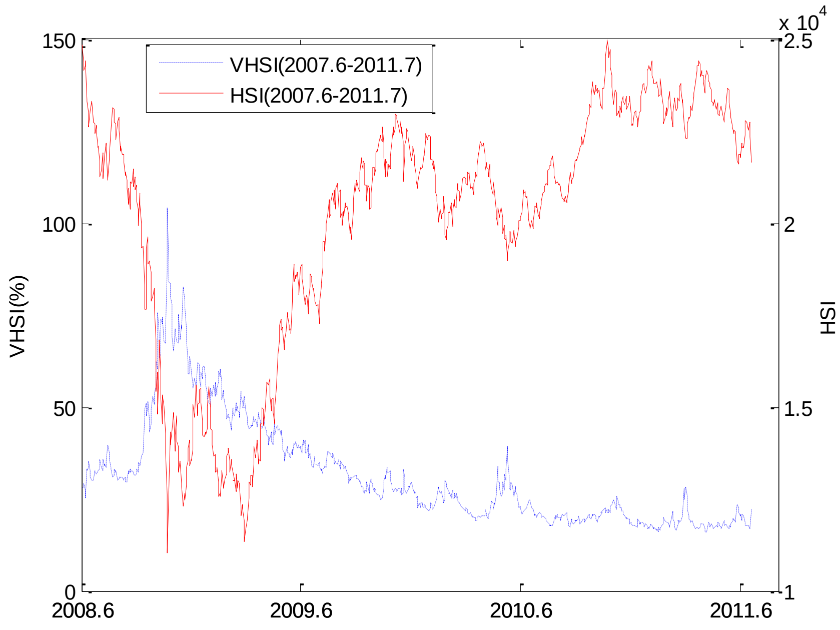

Fig. 7-1 Time Series of VHSI and HSI.

To numerically analyze the correlation between VHSI and HSI, we use the Pearson's correlation coefficient which defines the correlation as the covariance of two variables divided by the product of their standard deviation.

$$\rho_{X,Y} = \frac{\text{cov}(X,Y)}{\sigma_X \sigma_Y} = \frac{E\left[(X-\mu_X)(Y-\mu_Y)\right]}{\sigma_X \sigma_Y} \tag{7.2}$$

Here, in our analysis, X represents the series of VHSI, Y represents the series of HSI. We divide the time series data into four sections and calculate their respective correlation coefficients. Table. 7-1 shows that different sections have different time periods and the correlation coefficients are also different from each other. Besides, the number of observations affects the correlation coefficients.

Correlation coefficients of Sections 1 and 3 are relatively high. Their numbers of observations are also low. Section 4 also has a correlation of -0.5937 but its number of observations is high. As is known, observations over a long period of time may be unable to capture the trend of time series. Nevertheless, Section 2 has observations over a short period but its correlation is also low. That is because the trend of this section is fluctuating and unstable. The above analysis suggests that the correlation coefficient may be affected by two factors:

Table. 7-1 Correlation Coefficients between VHSI and HIS.

Section No.	Time Period	Correlation Coefficient	Number of Observations
1	Jun 2008 ~ Nov 2008	-0.9629	120
2	Dec 2008 ~ Mar 2009	0.0974	80
3	Apr 2009 ~ Nov 2009	-0.8858	160
4	Dec 2009 ~ Jul 2011	-0.5937	409

[1]. The number of observations affects the correlation coefficient. If the observation period is too large, the correlation coefficient is affected severely.

[2]. The trend of the underlying asset price should be monotonous. If the process of underlying asset price fluctuates during the observation period, the coefficient also changes dramatically.

Therefore, a suitable length of time period of observations and a stable time series trend are critical for obtaining high correlation coefficient. In the next section, we propose an adaptive correlation coefficient scheme to capture the correlation between the underlying price process and the volatility process.

7.3 Regression-based Correlation Coefficient

The traditional Heston model and its extension usually considered the correlation as constant variable. However, this assumption is unsuitable for real market. In this chapter, we use regression method to model the trend of underlying asset price and volatility. Since different time periods of underlying asset price and volatility own different series patterns, such as different parts of Fig. 7-1, the observation lengths of time periods are optimized in empirical tests section.

Luo and Zhang (2010) had presented to apply regression method to exam the relationship of realized volatility, VIX, realized variance, historical volatility and historical variance and to obtain the information content of volatility term structure. However, they only forecasted volatility rather than underlying asset price. Actually, we apply regression method to predict price by regressing the correlation of underlying asset price and volatility. Through different regression models of correlation coefficient, the predicted prices are derived from volatility stochastic process. The performance of different regression models is compared in Section 7.4.

7.3.1 Simple Regression

Different observed periods own different series patterns. The patterns are also affected by the observation period length. As is shown in Fig. 7-1, the series pattern of HSI from Mar 2009 to Jun 2009 is like a linear pattern with high correlation coefficient. So is the pattern of VHSI in the same observed period. Hence, we use a simple regression model to estimate the correlation of HSI and volatility as below.

$$S_t = \alpha_t + \beta_1 V_t + \varepsilon_t \tag{7.3}$$

where S_t is the underlying asset price; V_t is the stochastic volatility. α_t specifies the expected position of HSI. β_1 shows the unique effect of volatility on HSI. That is the expected change of HSI by a unit change of volatility.

7.3.2 Polynomial Regression

Some series patterns can be modeled by simple regression. However, this is not always the case. If we measure the correlation of HSI and volatility from Mar 2009 to Mar 2010, the linear regression is not suitable anymore. The series pattern is dropping down first and then climbing up. This series looks like a parabolic curve. Hence, we propose to use polynomial regression model to measure the correlation for this pattern.

$$S_t = \alpha_t + \beta_1 V_t + \beta_2 V_t^2 + \varepsilon_t \tag{7.4}$$

where β_2 means the degree of effect of V_t^2 on HSI. That is the expected change of HSI by a unit change of V_t^2. If β_1 is larger than β_2, this means V_t has a greater effect on HSI. If β_1 is less than β_2, V_t^2 contains more information and has a greater impact on HSI then V_t.

7.3.3 Autoregressive model

Although the polynomial regression model has a good performance for the parabola pattern, it is still not good enough for the vibrating pattern, such as the pattern from Oct 2010 to Jun 2011. The vibration mode is a more complicated pattern, which needs a complicated regression model. Since this pattern is vibrating belong a center line and has a mean-reversion characteristics, we propose to use Autoregressive model to evaluate this pattern as follows.

$$S_t = \alpha_t + \beta_1 V_t + \beta_2 S_{t-1} + \varepsilon_t \tag{7.5}$$

where β_2 means the effect of S_{t-1} playing on asset price.

These three different regression models are applied to different series patterns. Different observation periods may also affect the series patterns. Hence, for a given observation length, we compare the error term of these models and select the suitable model with smallest regressive error. Then, we propose the following equation to capture the correlation coefficient.

$$\rho = \frac{2}{\pi} arctg \beta \tag{7.6}$$

where ρ represents the correlation coefficient. β is the coefficient of regressive variant. For simple regressive model, β is equal to β_1. For polynomial regression, since the curvature reflects the instant change of correlation, β_2 is considered as β.

For auto-regressive model, β_1 is considered as β.

In the basic Heston model, the underlying asset price is determined by a stochastic process with a constant drift rate and stochastic volatility. Stochastic volatility is another stochastic process but has a correlation with the first process. Therefore, the underlying process is very similar to the Geometric Brownian process (GBP). In GBP, the underlying asset price process is a stochastic process with a constant drift rate and constant volatility. Stochastic differential equation (SDE) of GBP is as follows.

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{7.7}$$

where μ is the percentage drift rate. Basically, the drift rate is equal to the risk-free interest rate minus the stock dividend. σ is percent volatility. They are both assumed to be constant in the assumption of geometric Brownian movement. W_t is a standard Wiener process.

In our model, we have already incorporated the implied volatility term structure. The term structure of interest rate is also available from HIBOR[5]. For a given time t and initial stock price S_0, an analytic solution can be derived from SDE of (7.7) by applying the Ito's interpretation[6] (Ito, 1951).

$$S_{t+1} = S_t \exp\left(\left(\mu_t - \frac{V_t}{2}\right)t + \sqrt{V_t}W_t^S\right) \tag{7.8}$$

where S_t is the stock price at time t. Right hand side of (7.8) represents the calculation of future stock price based on the initial value. For a given time t, the implied volatility is obtained from the volatility term structure described above. μ_t is the drift rate, a function of time t.

7.4 Empirical Tests

In this section, the market option data from Hong Kong Exchanges and Clearing Limited (HKECL) is selected. The option data spans from Jan 2011 to Jun 2012 are selected for analysis. However, before we apply this data to do empirical tests, the market data should be cleaned from three aspects. First, the options with price near

[5] HIBOR is Hong Kong Interbank Offered Rate, the annualized offer rate that banks in Hong Kong offer for a specified period ranging from overnight to one year, normally including overnight, 1 week, 2 weeks, 1 month, 2 months, 3 months, 6 months and 1 year.

[6] In mathematics, a particular type of stochastic process applies the Ito's Lemma to calculate the differential of a function. It is named after Kiyoshi Ito, who discovered the lemma. It is the counterpart of stochastic calculus in ordinary calculus. It uses the Taylor series expansion by conserving the second order term in calculus. This lemma is widely used in finance and related fields. It is best known for its application for deriving the Black-Scholes option pricing model.

the money are selected. Since the implied volatilities of options with far out of the money are irrationally large, these volatilities will affect the regression models and make noise to empirical tests. Second, the options with bid price small or equal to ask price are selected. The option data with bid price larger than ask price is unreasonable. In the real market, the selling price should always large or equal to buying price. No one wants to buy at a high price rather than at a low price. Hence, we consider this type of option data as noise data and eliminate them. Finally, the option prices with transaction volume are selected. Although some options own both bid price and ask price, there is no trading volume. These option prices are useless for tests.

The empirical tests contain in-sample data training and out of sample test. There are total 18 months of option data. The implied volatilities of these data are calculated from these data. The weight implied volatility of each day during the testing period are computed according to a fix weighted scheme. We use a period of 12 months to train Heston model with dynamic correlation. The volatility and underlying asset price are regressed according to three different models above. The regression training period is a key parameter. If it is too small, the polynomial efficient can be solved by regression. If it is too large, the data pattern will become complicated and affect regression results. Hence, we choose to search the optimal observation period from 3 days to 3 months.

Since different regressive models are suitable for different data series pattern, they need different observation periods. Hence, three regressive models are trained, respectively. The correlation coefficient is calculated from the regression results according to equation (7.6). According to Heston (1993), drift rate of stochastic process is equal to the interest rate minus dividend, e.g. $\mu_t = r_t - D_t$. W_t^S is a Wiener process, which is generated by standard normal distribution. However, since W_t^S has an optimal correlation with W_t^V, they are calculated as follows.

$$W_t^V = \sqrt{dt}\varepsilon$$

$$W_t^S = \rho W_t^V + \sqrt{1-\rho^2}\sqrt{dt}\varepsilon \tag{7.9}$$

where dt is the time interval of term structure (measured in year); ε is the standard normal distribution variable; ρ is the correlation of these two processes. Similarly, stochastic process of implied volatility based on term structure is generated as follows.

$$V_{t+1} = V_t e^{\left(\kappa(\theta - V_t) - \frac{\sigma^2}{2}\right)dt/V_t + \frac{\sigma W_t^V}{\sqrt{V_t}}} \tag{7.10}$$

where κ is the mean reversion speed; θ is the long run mean level of implied volatility which can be obtained from term structure; σ is volatility of volatility. These parameters can be obtained from historical options data.

For out-of-sample tests, we use the data spanning from Jan 2012 to Jun 2012, totally 6 months, to test the prediction performance of three regression models. One-day-ahead prediction is the forecasting of underlying asset price by Heston model for one day. Five-day-ahead means to predict underlying asset price for five days. One-month-ahead is to forecast for a period of one month. Besides, we also search for the optimal testing period for different regression models. As is shown in Table. 7-2, the optimal regressive period is 9 days. The effect of volatility playing on underlying asset price is -2.16, which is -0.724 changed by equation (7.8) to correlation coefficient.

Table. 7-2 Efficient Comparison of Regression Models.

Model \ Efficient	α	β_1	β_2	ρ	Period
Simple Regression	19451.5	-2.16	-	-0.724	9
Polynomial Regression	18568.3	-0.23	-1.97	-0.701	20
Auto-Regression	12732.6	-1.66	0.32	-0.655	42

From Table. 7-2, we find that the correlation of simple regression is larger than the correlation of other two models. This means that the simple regression model can better capture the correlation of underlying asset price stochastic process and volatility stochastic process than the other two regression models. For polynomial Regression model, the square of volatility has a higher correlation with underlying asset price than volatility. The efficient of square of volatility also shows the instant changing speed of volatility. For auto-regression, it shows that the underlying asset price has autocorrelation. However, this autocorrelation affects the effect of volatility playing on underlying asset price.

Table. 7-3 Prediction Performance Comparison of Regression Models.

Model \ Performance	One-Day-Ahead	Five-Day-Ahead	One-Month-Ahead
Simple Regression	56.1%	53.7%	51.3%
Polynomial Regression	55.4%	52.3%	51.7%
Auto-Regression	53.4%	50.9%	48.9%

Table. 7-3 shows the prediction performance comparison of regressive models. Simple Regression model has One-Day-Ahead prediction performance of 56.1%, which is better than the other two models. This is because simple regression has a small regressive period and can capture the correlation preciously. Since the weighted implied volatility reflects the future view of market participants, this volatility owns some implied information content which can tell what future market will be. When simple regression model accurately capture this correlation, it then can predict more preciously than the other two models. By comparing the other two prediction standards, we find that simple regression model has a better performance.

7.5 Conclusions

For the study of Heston model, most of literatures are focusing on multivariate stochastic process. Although some studies touched the topic of considering the correlation of Heston model as stochastic process, this method is complicated and time-consumption. So this method is unsuitable for real time calculation. However, we propose to use regression model to estimate the correlation of underlying asset price and volatility. The volatility is calculated from implied volatility with a fix weighted scheme. Besides, three different volatility models are used to estimate the correlation. From our empirical tests, we find that the simple regression model outperform the other two regression models.

Chapter 8 Index Option Strategies Comparison and Self-Risk Management

VaR(Value at Risk) and CVaR(Conditional Value at Risk) are the criteria which are commonly used to evaluate the potential risk to investors. For the risk management of option, buying or selling the underlying assets is useful for hedging the potential risk in a certain degree. But as to the index option, this method does not work for the difficult of absolutely hedging the moving of index. Fortunately, it is possible for investors to hold the risk in a certain range by using option itself which is called self-risk management in this chapter.

8.1 Introduction

Option plays a significant role in financial markets and it is widely used in both arbitrage and risk management. As one kind of financial derivatives, it is an instrument that specifies a contract between two parties for a future sale or purchase of an asset at an agreed price. The buyer of an option gets the right, but not the obligation, to engage in that transaction, while the seller is bound to fulfill the corresponding obligation.

Because of some special features such as the leverage, it can perform well in investment for both arbitrage and risk management. A lot of works have been done to analyze the function of option in changing the return distribution and controlling the risk. Especially the put option can always truncate the downside risk of the underlying asset efficiently and the covered call is also useful to reduce the volatility of return. Even though, option is used as a tool of arbitrage more than used as a tool of risk management. Compared with the property such as stock, fund, bond and so on, the investment on option usually facing higher risk.

Regrettably, most of work focused on the features of option on risk management more than to control the risk of option itself. The risk management of option is usually more difficult than using option to control risk. As to the equity option, by buying or selling the combination of underlying asset and option, the risk can be controlled in a certain degree, but as to the index option it is hard to control the risk well in this way, because there is no production which can hedge the movement of the index. Though there is some production such as index future which can hedge partially, it is hard to control dynamically. Of course, there is a lot of research which analyzed how to control risk of option dynamically, such as the research about hedging the parameters of Greeks. It can work in theoretical in most of time, but in reality it is impossible to keep the state dynamically and it is very complex to construct this kind of portfolio.

As to this kind of problem, this chapter wants to make new insights to the option strategies to analyze the potential risk of different strategies. Theoretically, the risk of the writer of option is unlimited while the buyer is limited. This characteristic is used for some investors to truncate the downside risk and it is the same by using this characteristic, many strategies can be constructed to meet the demands of the investors when they want to invest in different condition such as bear, bull, straddle, strangle, butterfly, condor and so on. Some of the combination can make the risk of investor limited and of course it will lead to a cost to do that. The advantages of this method is that the procedure of doing risk management can be done by using option it is not needed to introduce the other productions and the risk can be hold on a range. So by using this kind of strategies, it is called self-risk management.

What the investors usually care about is the expected return and the risk they may afford. So expected return and some risk criteria are used in this chapter when the strategies are selected. VaR (value at risk) and CVaR (conditional value at risk) are the criteria which are usually used in reality. By trading at Hong Kong market, we know that as to the HSI option, there are thousands contracts traded every day and almost all of contracts which are traded are at the money options. And only the at-the-money option has liquidity. Once some statistical methods have been used to analyze the return of option strategies, and some results pointed out that selling out the money put always has higher return when using the option in arbitrage. But this is not enough, the movement of the Hang Seng Index is rapidly and the single position always leading to a huge lost. Even some investors get their return by construct the portfolio to get the time value, the method to construct the portfolio can make important influence to the return. There are dozens of option strategies and there are hundreds of options which can be used to construct these strategies.

Most of previous works are doing the statistical research to analyze the probability of get profit by using these strategies. This chapter will choose expected return, VaR and CvaR as the criteria to choose the option strategies under the assumption that the movement of the Hang Seng Index following the geometric Brownian motion and then to analyze the performance of different strategies in getting profit and managing the risk.

By doing this work, the results can be the references when investor constructs the portfolio in different conditions. And by calculating the VaR and CvaR of these strategies, it is helpful for investor to know the potential risk and control the risk better. Compared with the method of setting the stop points which is usually used in the trading of index option, making these new insights to these strategies can help to construct the portfolio which can hold the risk in a range. It means that investors can build the position that the risk can be pre-calculated. As to these strategies, the different positions can hedge the risk each other so it can be called self-risk management.

8.2 Review

Since it was first traded unified and standardized in CBOE in April 1973, the instrument of option has developed significantly over the past few decades both in risk management and arbitrage. When Black and Scholes (1973) created the model to price the option in their paper "The Pricing of Options and Corporate Liabilities", trade in options has been booming. The model is called Black-Scholes model and is still widely used. The most important functions of option are risk management and arbitrage. Papers by Ross (1976), Breeden and Litzenberger (1978) and Arditti and Kose (1980) pointed out that option spans opportunities which increase the efficiency of financial market. Bookstaber and Clarke (1983, 1984) use the method of Monte Carlo simulation test the influence of the option can make to change the return distribution of portfolio. They point out that early study in this area is that of Merton et al. (1978, 1982). But most of these research focuses on the function of equity option in spanning the market opportunities. However, since 1973, Fish Black and Myron Scholes creating the model to price of the option, option is mainly used in two aspects arbitrage and risk management reality. And as investigation in 2002 in Hong Kong market these two parts account for 41 percent and 46 percent.

Based on this, it is very important to make insights to the complex option strategies which are used for arbitrage. Many books have been written to illustrate the characteristics of option strategies (Lin, 2010). But only the method to construct these strategies is not enough for investors. Joshua and Tyler (2001) analyzed the return of strategies single call, put and zero-delta straddle in detailed by using the data of s&p500 in their paper "Expected option return". In this chapter, they analyze the expected return of call, put and zero-delta straddle by divided them as different strike prices, at-the-money and out-of-the-money. And they also use the result to give the return sensitivity to different parameters such as volatility, risk-free interest rate and so on. It is very helpful for investor to learn about the characteristics of these kinds of strategies and it makes sense to the trading in reality. But this is not enough, to complete the using of the trading strategies in index option market. This chapter will use the criteria of expected return, value at risk and conditional value at risk to make new insights to the option strategies which are constructed by one single position, two positions even three or four position. And expected return is the profit the investor expected to get. VaR and CVaR are important criteria which are usually used to evaluate the potential risk the investor may bear. Ahn et al. (1997) once used the VaR to evaluate the performance when using the option in risk management in their working paper "Optimal Risk Management Using Options".

Though using option for arbitrage has been very popular and its share in financial markets has been growing, almost all financial organs design their strategies empirically. In reality, investors trust their experience more than quantitative analysis. In this condition, the research in this chapter is very significant. The results can be used as a guide when investors use the different option to construct the portfolio and it is can help the investors to pre-calculate the risk they may face and construct the portfolio in a certain range of risk.

This chapter uses the option of HSI as an example. The predicting data in this chapter is the result of a distortion of Monte Carlo simulation. Using the Monte Carlo simulation simulates the moving of the HSI, and then choose the at the money options to construct the different strategies. As to any one kind of strategy, it can be ranked by the criteria expected return (ER), value at risk (VaR) and conditional value at risk (CvaR) and as to any kind of strategy, only the strategy which has good performance is chosen to be used to compare the different performance of each kind of strategy.

This chapter is composed of four main parts. The first part and the second part are the introduction which outlines the background, describes previous research works in the related field and the significance of analyzing the performance of different strategies. The third part is the model and the process to compare the different strategies. In the last part, the result is discussed and the conclusion which summarizes the significance of this chapter and the meaning of the model proposed in this chapter will be given.

8.3 Theoretical Background and Model

The model proposed in this chapter is based on the HSI (Heng Seng Index), implying that the option strategies yielding profit depends on the trend of HSI. The option is European style option which has an expiration date, T, and it can only be executed at the expiration date but it can be traded at the market as a kind of merchandise. It has been mentioned that investors who use option strategies for arbitrage usually try to find the option which strays away from its normal value and earn the difference in value by buying and selling the option. There are dozens of option strategies which can be used for arbitrage and it is impossible to list all option strategies in one paper. However, according to one kind of criteria, one of any kind of strategies can be chosen as an example to analyze this kind of strategy.

To evaluate the performance of these kinds of strategies, the movement of the Hang Seng Index is need to be simulated. The most commonly used method is Monte Carlo Simulation. Monte Carlo Simulation is used to simulate the movement of the HSI, then based on the simulated movement of HSI, the at the money index options can be chosen to construct the different strategies, such as call and put which are single position, bull, bear, straddle and strangle which are the strategies constructed by two positions, even the strategies of butterfly and condor. The return of these strategies is based on the result of the simulation, so one time simulation is not enough to analyze the regular pattern of these kinds of strategies. So different starting times are chosen to do the experiments and as to each experiments hundreds times of Monte Carlo Simulation are done to calculate the return and risk of different strategies.

The first step, we choose the classical Monte Carlo Simulation which is usually used in the financial area:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dz_t \tag{8.1}$$

In this equation, S_t is the price of the underlying asset at the time t, and μ is the mean drift value while σ is the volatility and z_t is the geometric Brownian motion. As to the problem in this chapter, S_t can be viewed as HSI and μ can use the value of risk free rate γ and σ can be replaced by the volatility of HSI. Then by using this equation, we can simulate the motion of the HSI. The results of simulation are used to test the performance of index option strategies. Eight kinds of strategies are analyze in this chapter and it includes call, put, bull, bear, straddle, strangle, butterfly and condor.

By using the simulation results, all of these strategies can be applied into the virtual trading. The parameters of expected return (EP), value at risk (VaR) and conditional value at risk (CVaR) can be calculated to evaluate the performance of these strategies in this kind of virtual trading. The price of the index option in these simulations can be calculated by applying the Black-Scholes model which is the classical option pricing model in both reality and theoretical area. The equation of Black-Scholes which is used to calculate the price of index option can be expressed as:

$$[p_c^{t'}, p_p^{t'}] = blsprice(S_{t'}, K, r.T - t'.v, y) \tag{8.2}$$

where the parameters $p_c^{t'}$ and $p_p^{t'}$ are the price of the corresponding call and put option at the time t', and the corresponding value of underlying asset is $S_{t'}$, in This chapter it is the Hang Seng Index. The strike price are K and the maturity time is T. r and v are corresponding risk-free interest rate and volatility. y is annualized, continuously compounded yield of the underlying asset over the life of the option, expressed as a decimal number. In this chapter the default number is zero.

The simulation trading can be done by combining the part of Monte Carlo Simulation and Black-Scholes model. As to the index options of HSI which are traded in the Hong Kong market, only the at-the-money options have the price and their liquidity is well for arbitrage. As to the out-of-the-money options, there usually are not the price in reality. Even they have price, the liquidity is so bad for arbitrage and it is hard for the investors to stop their position. Adding this kind of options will lead to the results far from the real performance in reality. To follow the reality, only the strike prices of the options around 8 percent of the HSI are selected for the simulation trading. It can almost cover all the volume in Hang Seng market. Then for to evaluate the performance of the strategies which have been mentioned above, expected return, value at risk and conditional value at risk need to be calculated for each time of simulation.

$$EP = \left(\sum_{n=1}^{N} \left(V_{t'}\left(S_{t'}^{n}, K, t'\right) - V_{t}\left(S_{t}^{n}, K, t\right) \right) \right) / N$$
$$VaR = V_{t'}\left(S_{t'} \bullet \theta(\alpha), K, t'\right) - V_{t}\left(S_{t} \bullet \theta(\alpha), K, t\right) \quad (8.3)$$
$$CVaR = V_{t'}\left(\beta\left(S_{t'} \bullet \theta(\alpha)\right), K, t'\right) - V_{t}\left(\beta\left(S_{t} \bullet \theta(\alpha)\right), K, t\right)$$

In these equations, N means the times of Monte Carlo Simulation and K is the strike price. For the strategies which are constructed by more than one position, K is a vector. $\Delta t = t' - t$ is the holding period where t is the time to build the position while t' is the time to stop the position. The function V is used to calculate the value of the position. The function $\theta(\alpha)$ is a mapping function following by the definition of value at risk with boundary α which can find the corresponding value of underlying asset to the specific strategies for those the probability of leading to the biggest loss of a value will not exceed $1-\alpha$. And as to the function β, it is a cumulative function. It is following the definition of conditional value at risk. The function of β is a weighted average between the value at risk and losses exceeding the value at risk. So the CvaR is usually less or equal to the value of VaR. In this chapter the probability α is defined as five percent.

The eight strategies which have been mentioned above will be calculated and these eight strategies almost covered all strategies which are usually used. And these eight strategies include one position strategies such as single call or put and two positions strategies such as bull, bear, straddle and strangle, even three and four positions strategies such as butterfly and condor. And these strategies also cover the strategies of upward trend, downward trend and fluctuation.

8.4 Results and Analysis

Based on the illustration above, the simulation trading can be done and the results will be analyzed in this part. In the following of this part, two experiments are done to do this analysis and sixteen strategies are chosen to be analyzed including long call, short call, long put, short put, bull call, bull put, bear call, bear put, long straddle, short straddle, long strangle, short strangle, long butterfly, short butterfly, long condor and short condor. And all of these strategies are numbered from one to sixteen by order.

Two experiments are done while the first experiments from 12th Dec. 2011 and the second from 12th Fab. 2012. The time periods of the two experiments are both 100 days. And for each of these two experiments, 100 times of Monte Carlo simulation was done. The figure 1 shows the mean value of HSI of these two experiments and the dashed line is the trend of the first experiment which started to do the Monte Carlo simulation from12th Dec. 2011 while the real line to the second experiment. The ordinate axis is the HSI and the cross shaft is the date. Compared with the first trend, the second line has an obvious trend in the next 100 days. It means that the second experiment has the higher volatility. These two experiments include both the fluctuant market and the clear trend market.

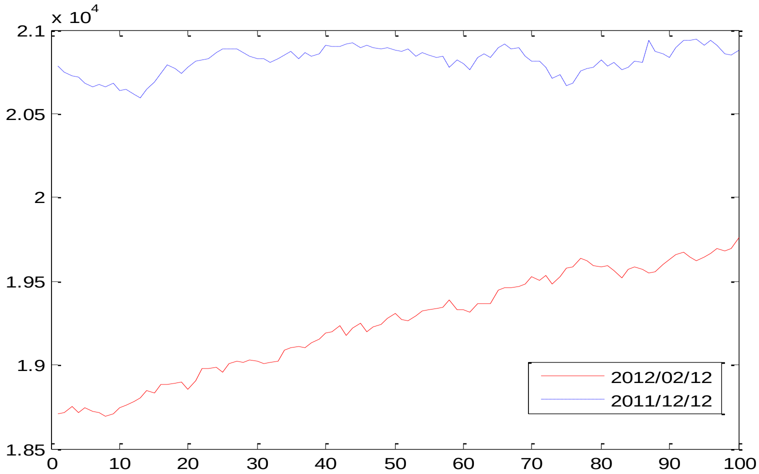

Fig. 8-1 The mean value of Monte Carlo Simulation.

The two experiments have the same process and model and what are different are the parameters of the simulation and the results. By doing the simulation, a series of distribution of Hang Seng index can be got, and then based on the distribution, the price of the index option can be calculated and the price is also a distribution. By compared with the price of starting day, the return of each strategy can be calculated. As to the strategies with more than one position, all kinds of combination are calculated. The performance of each strategy can be evaluated.

To evaluate these strategies, three kinds of criteria are selected which include the expected return (EP), value at risk (VaR) and conditional value at risk (CVaR). The variances corresponding to these criteria also can be calculated to test the stable of these strategies.

The results are showed in the tables of appendix. The first table is the result corresponding to the date of 12th Dec. 2011 while the second table to the date 12th Fab 2012. The two tables have the same contractures. The sixteen columns represent these sixteen strategies in the order we mentioned above. And both of these two tables have six rows and the first, third and fifth row represents the EP, VaR and CvaR while the even rows are the variances corresponding to the EP, VaR and CvaR.

To analyze the performance intuitive the parameters of EP, EP's variance, the VaR and the VaR's variance of these sixteen strategies are showed in the following figures. And we will make some analysis based on the results which are showed in these figures. Because the results of CvaR are very similar to the results of VaR, the data can be found in the appendix. So it will not be analyzed in this part.

Fig. 8-2 shows the expected return of these strategies in these two experiments. The horizontal axis from one to sixteen represents the sixteen strategies and the order corresponds to the order we mentioned above. In this figure, the dash line represents the first experiment while the real line represents the second.

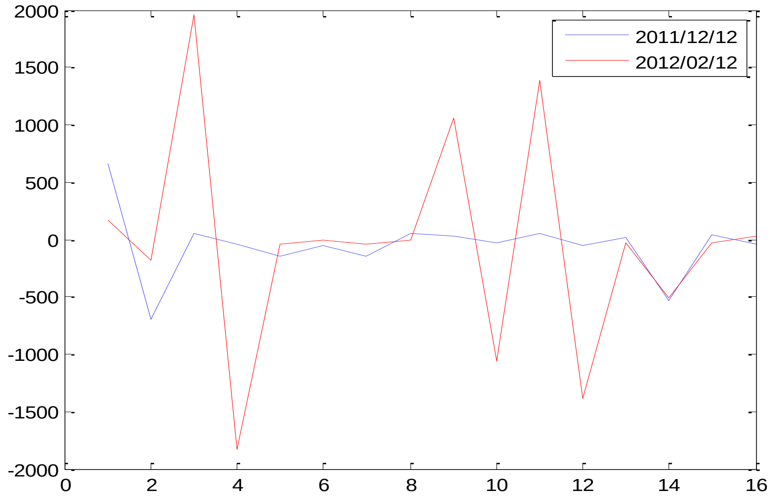

Fig. 8-2 The mean expected return of these sixteen strategies.

Fig. 8-2 keeps in accordance with the reality very well where the big win and the big loss always happens when there is a big trend in index. And the odd number strategies and the even number strategies are opponent usually.

As to the sixteen strategies, the first four strategies usually have big win or big lost when there is an obvious trend in index, because there is only one position. And the strategies of straddle and strangle has the same characteristics with the single position but the swing is a little small than the single position strategies. The strategies such as bull call, bull put, bear call, bear put, butterfly and condor usually have small swing. And these characteristic also can get from the following figure 3, the variance of expected return of these sixteen strategies.

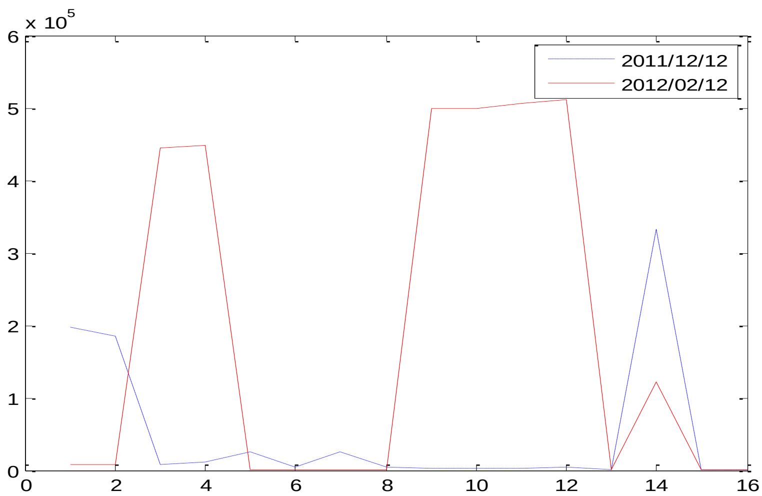

Fig. 8-3 The variance of expected return of these sixteen strategies.

From Fig. 8-3, we can get the conclusion that as to these strategies, the return to the strategies bull, bear, butterfly and condor usually has lower variance especially for the market which has an obvious trend. But to the strategies straddle, strangle and the strategies which have only one position, the variance is huge. It means that these kinds of strategies bring high profit which accompany with high risk. The similar conclusion also can be got from the analysis of the VaR and CvaR. As the former strategies, it seems that these kinds of strategies can resist the risk their own in a certain degree especially in big market. So in this chapter, this performance can be viewed as the function of self-risk management. These kinds of strategies maybe not bring a huge profit in one transaction and they also will not leading to a huge loss. So if investors don’t have a big confidence with the future of the market, these kinds of strategies may be a good choice. And I think it can provide a good chance for the risk averse to invest on the index option.

The following two figures which are similar to the two figures above also can get the similar conclusions above. But these two figures are the results about the value at risk. As to the results of conditional value at risk, because it is very similar to the result of VaR, so we will not analyze the CvaR singly and only give the results in the appendix.

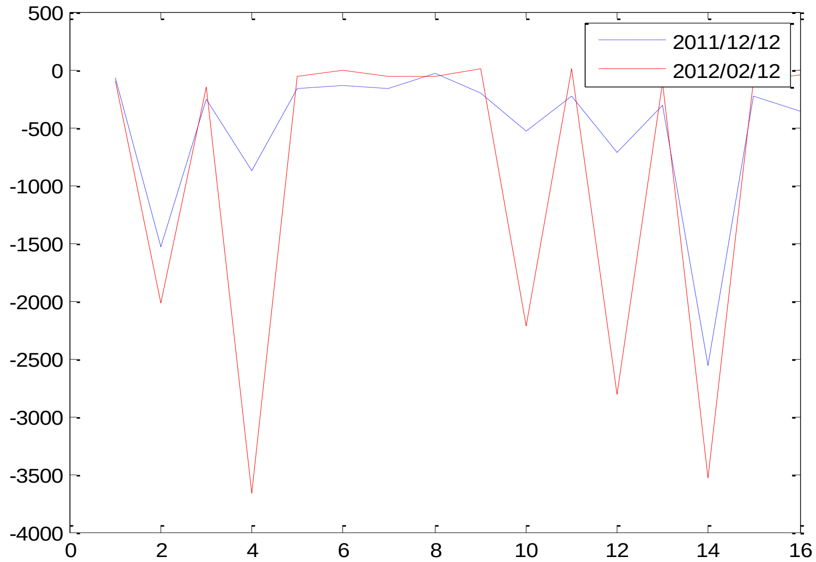

Fig. 8-4 The value at risk of these sixteen strategies.

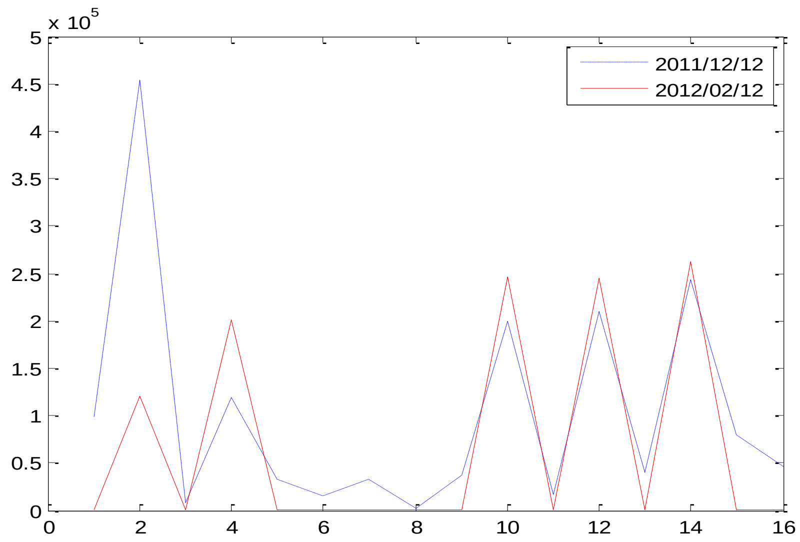

Fig. 8-5 The variance of value at risk of these sixteen strategies

Fig. 8-4 is the mean value of VaR of these strategies while Fig. 8-5 gives the variance of the value of VaR. As the same as the conclusion above, the strategies of bull and bear can hold the risk well and the variance of VaR of these strategies is also small. It means whether the market has a huge trend of fluctuation, the performance of these strategies is stable. But the difference is that though the strategies of butterfly and condor also have the characteristic of self-risk management. They don't perform well in criteria VaR and CvaR. Because these two kinds of strategies don't perform well both in the condition that the market has a big trend and fluctuation, we guess that because this kind of strategies have many positions and these positions accumulate the risk which make the VaR and CvaR not perform well.

So in conclusion, the single position strategies of index option are high risk high return. The strategies of straddle and strangle have the same characteristics with single position strategies. The strategies of bull and bear perform stable both in the condition that the market with an obvious trend and fluctuation. The return is stable and they also can control the risk well. As to the strategies of butterfly and condor the expected return is also stable but there still have the higher risk which accompany with the many position. All of these conclusions keep in accordance with the theory the contracture of these strategies.

This chapter is the first time to use the simulation method to analyze the characteristics of different strategies and it is the first time to make insides the statistic rules of these strategies. The research in this chapter almost includes all the index strategies which are usually used in reality. The research covers the strategies from one position to four positions. What's more, the two experiments in this chapter are different while the one is the market with an obvious trend and the other one is the fluctuation. It is significant for investor to make inside to the trading of index option. And it analyzes the function of different strategies in self-risk management. Three criteria are chosen to analyze these strategies. The expected return represents the ability of these strategies to get the profit while the VaR and CvaR are used to measure the risk the investor facing when the get the corresponding profit.

The disadvantages of this research are that the option data getting from the Hang Seng market, so the results are only suitable for the Hang Seng market. And because of the constrain of time to calculate the price of option by using the Black-Scholes model, the times of Monte Carlo simulation is not many enough to get more information and to get the more accurate results. The other aspect of the disadvantages is that the value of index option is calculated by using the B-S model, so it is impossible to avoid the mispricing of the model to the reality. What's more, this chapter only gives the characteristics and the rules of these strategies. It doesn't give the recommendations to the investors. Maybe in the future, if there is a method which can predict the index distribution, by combining the method in this chapter, the investment recommendations can be gave to the investors.

8.5 Conclusions

This chapter is the first to analyze index option strategies characteristics quantificationally, using evaluation criteria, to support investors making insides well to these strategies when using options for arbitrage. And by using the Monte Carlo simulation method to calculate the distribution of Hang Seng Index, the statistical results are calculated. By analyzing these results, they keep in accordance with the theory of the construction of these strategies. The results state that the single position strategies of index option are high risk high return. The strategies of straddle and strangle have the same characteristics with single position strategies.

The strategies of bull and bear perform stable both in the condition that the market with an obvious trend and fluctuation. The return is stable and they also can control the risk well. As to the strategies of butterfly and condor the expected return is also stable but there still have the higher risk which accompany with the many position. And the quantized results are given in the appendix. And the intuitive results and conclusion are given above. It is helpful for the investors to invest on the index option, because it not only make the analysis to the expected return but also analyze the ability of these function in risk management. Because of the difficulty in hedging the risk of the index option, the self-risk management is very important to the investors who are risk averse. What's more the expected return and the risk of the strategies are the most important to the investors when they invest. So making new insight and analyzing the index strategies quantitatively make great significant to the reality trading in index option. So this chapter makes sense both in theoretical circles and reality trading.

Appendix

Table 1. The mean value of expected return, value at risk, conditional value at risk and the variance of these three criteria base on the Monte Carlo simulation from the date 2011/12/12 to the 100days of future.

658.831	-698.781	52.71546	-43.9642	-143.499	-53.1794	-145.882	54.10272	31.65849	-31.6585	49.05539	-52.3391	17.77206	-531.9	36.06751	-36.2274
19669.94	18500.73	837.3808	1140.378	2607.711	422.526	2616.245	426.2803	196.8721	196.8721	273.2593	373.1577	16.80448	33292.8	76.644	77.39374
-70.993	-1530.15	-258.43	-875.565	-164.849	-141.58	-164.849	-26.4358	-208.246	-532.709	-223.775	-723.497	-304.954	-2558.18	-236.142	-364.798
981.8183	454.9192	85.93595	1197.329	321.5089	154.6704	321.5089	156.0981	372.0767	2000.059	171.6522	2098.453	404.6398	2433.08	801.9038	458.2672
-135.013	-1851.65	-276.431	-1258.41	-166.779	-148.774	-166.779	-37.7429	-213.066	-742.999	-228.067	-1037.08	-316.512	-3180.8	-250.113	-375.673
825.2752	4923.256	93.21392	2331.905	330.8407	176.7933	330.8407	17.37526	390.6867	3820.679	17.4402	4251.459	429.8912	4042.206	870.9459	429.2435

Table 2. The mean value of expected return, value at risk, conditional value at risk and the variance of these three criteria base on the Monte Carlo simulation from the date 2012/02/12 to the 100days of future.

163.9715	-186.518	1959.297	-1825.41	-46.0866	-1.17287	-46.1293	-4.89665	1057.809	-1057.81	1387.095	-1391.27	-28.6959	-509.269	-34.9971	34.99712
8342.47	8461.278	444741.1	447715.3	793.0253	122.4277	795.3619	89.22017	498475.9	498475.9	506241.1	510973.7	415.6132	121679.3	679.32	679.32
-97.7712	-2023.96	-145.267	-3658.01	-64.211	-11.6422	-64.211	-59.4784	12.3788	-2218.64	3.530885	-2807.49	-116.928	-3536.58	-104.63	-48.9318
193.8093	120935.2	302.199	201222.7	164.2511	9.652918	164.2511	100.9211	68.77803	245925.9	55.07265	245513	266.8005	261963.5	235.7471	73.69592
-98.5935	-4463.02	-154.342	-3749.57	-65.9956	-13.5177	-65.9956	-64.1349	-20.759	-2789.16	-29.7795	-3541.12	-123.059	-4599.37	-110.219	-60.8307
196.9691	347033.8	327.0306	215251.6	172.0292	10.88545	172.0292	110.2686	47.29136	396438.6	41.79015	395429.3	293.6506	441731.4	257.7949	88.73688

Chapter 9 Call-Put Term Structure Spread-based HSI Analysis

The HSI Volatility Index ("VHSI") aims to measure the 30-calendar-day expected volatility of the Hang Seng Index ("HSI"). It is derived from HSI put options and HSI call options in the two nearest-term expiration months in order to bracket a 30-calendar-day period. From the long-term regular, the trends of HSI and VHSI have intensive negative correlation and the VHSI is calculated from the implied volatility of out-of-the-money option which means the implied volatility of call and put options can reflect some information of the trend of HSI. Usually, the call and put options have different performance when the HSI goes upwards and downwards. So if the volatilities can be calculated separately and then analyze the different changes of call-put spread, it may form a good indicator to the motion of HSI. Based on these theories, this chapter tries to calculate and draws the curves of implied volatility of call and put options separately. The curves of implied volatilities of both call and put can be drawn and the distance between them which is called "call-put spread" can be calculated.

9.1 Introduction

The methodology of the VHSI is based on the CBOE Volatility Index ("VIX") in the US market. Modifications have been made to take into account trading characteristics of HSI options in the Hong Kong market. As to the Hong Kong market, for each of the near-term and next-term options, out-of-the-money call options with strike prices higher than or equal to but lower than or equal to 120% of HSI are selected. Out-of-the-money put options with strike prices lower than but higher than or equal to 80% of HSI are selected. And the methodology has been published by Hang Seng Bank. According to the long-term observation, the trend of VHSI has a significant negative correlation with the trend of HSI, and this result of the observation has been generally accepted by the industry. That means the VHSI contains much information which has a relationship with HSI. According to the statement of Hang Seng Bank, the VHSI is calculated by combining implied volatility of call and put option. So the implied volatility of options can reflect the motion of HSI and Andersen et al. have done the related worked that forward looking market-based implied volatility in 2003. We know that call options and put options have different behaviors whenever the HSI goes upwards or downwards. Inspired by this result, this chapter tries to calculate and combine the implied volatility of call and put options separately and by analyzing the spread between the call volatility and put volatility to predict the trend of HSI. The implied volatility of call and put options is calculated by using the mainstream method, Black-Scholes Model, which is widely used in financial engineering.

By using B-S model, once the parameters of price (underlying asset), strike price, rate, time to maturity, the value of the option and the style (call or put) are known, the implied volatility can be calculated. Implied volatility, a forward-looking measure, differs from historical volatility because the latter is calculated from known past returns of a security. Because of the importance of implied volatility, many scholars have done related work to make insights to the implied volatility and they make great contributions to the development of research in the field of implied volatility (Andersen et al., 2003; Bollerslev and Zhou, 2002, 2003; Bollerslev et al., 2007). Most of their works are focused on how to calculate the implied volatility accuracy but not to use the information contained in the implied to predict the trend of the market. Standard deviations implied in option prices are used as predictors of future stock price variability in 1981 (Beckers, 1981). Emanuel and Kani (1994a, 1994b), Dupire (1994), and Mark (1994) hypothesize that asset return volatility is a deterministic function of asset price and time, and develop a deterministic volatility function (DVF) option valuation model that has the potential of fitting the observed cross section of option prices exactly. But it is no better than an ad hoc procedure that merely smooth Black–Scholes (Black and Scholes, 1973) implied volatilities across exercise prices and times to expiration (Dumas et al., 1998). So in this chapter the Black-Scholes model is used to calculate the implied volatility and the implied volatility will be used as predictors of the trend of HSI.

The idea has been mentioned above that is based on the different performance of implied volatility of call and put options and the intensive negative relationship between VHSI and HSI, the VHSI will be divided into the volatilities of call options and put options. And then both call options and put options are divided into different group by expired data. Because there almost are no deals of the options which have long time to maturity, only the options which will be expired in recent months are selected. The mean values of the same style options which have the same expired date can be calculated. Then two curves can be got. The first one is the curve of the mean value of implied volatility of call options while the second one is put options. And these two curves always change with the motion of HSI. There is a lot of information in these two curves especially the interspace and the relative positions of them. When there is a downward trend of the market, the implied volatility of put options is usually higher while there is an upward trend, the implied volatility of call options higher. So we try to build the model to test the relationship between the implied volatility and the trend of Hang Seng market. Then the spread between implied volatility curve of call and put options is calculated as the indicator to predict the trend of market. In this chapter, the Hang Seng Index options are chosen to calculate these two curves and the time period is 15 minutes. That means in every 15 minutes all the data of options which have volume are calculated to predict the trend of next

15 minutes. The data of one year, form April 2011 to April 2012, is selected to build the model to predict the trend of HSI.

This chapter is composed of four main parts. The first part is the introduction which outlines the background, describes previous research works in related field and the reason to do research about this field. The second part is the theoretical background and technology which will be used in the model and the process of prediction. In the third part, the result will be given to prove the efficiency of the model in predicting the HSI. In the last part, the result is discussed and the conclusion which summarizes the significance of this chapter and the meaning of the model proposed in this chapter will be given.

9.2 Theoretical Background and Model

9.2.1 Data

The data in this chapter is collected for Hong Kong market from April 2011 to April 2012. And the time period is 15 minutes which means the data is collected from the market every 15 minutes and all of the data of the options which have prices in the 15 minutes are collected including the value of the options, style, expire date and so on. The risk free rate which is corresponding to the interest rate and the corresponding Hang Seng Index which will be used to test the efficiency of the model are all from the real data of HK market. As we have mentioned above, as to the B-S model, once the parameters of price (underlying asset), strike price, rate, time to maturity, the value of the options and the style (call or put) are known, the implied volatility can be calculated by using B-S model. Based on the data we collect, the implied volatility of each option which has price can be calculated and these volatilities are the input parameters for the prediction model.

9.2.2 Theoretical Background

As to the initial data, it is used to be calculated the corresponding volatilities. Berestycki et al. (2004) stated the method to calculate the implied volatility in their paper “Computing the Implied Volatility in Stochastic Volatility Models”. And many scholars have done a lot of works on the field of implied volatility (Lee, 2004; Shehata and Mickaiel, 2012). As known and mentioned above, Black-Scholes model is the mainstream option pricing model and as to B-S model, any one of the all parameters can be calculated once the other parameters are known. So the equation to calculate the implied volatility can be written as.

$$IV = blsiv(P, S, r, T-t, v, y, B)$$

And in this equation, IV is implied volatility which is needed to be calculated, and $blsiv(\bullet)$ is the B-S function which is used to calculate IV. The parameter P is the price of the underlying asset. As to the Hang Seng index option, it can be expressed as “HSI”. S is the strike price of the corresponding option. T is the expire date while t is the time now. Then the meaning of the parameter $T-t$ is the time to maturity. v is the value of the corresponding option and y is annualized, continuously compounded yield of the underlying asset. Here the default value for y is zero. B is a binary parameter, call or put, which states the style of option.

VHSI which is known as a combination of the implied volatilities of out-of-the-money call options and put options has intensive negative relationship with HSI. It proves that implied volatility contains information of the trend of HSI. But we know that implied volatilities of call options and put options usually have different changes whenever the Hong Kong market goes upwards or downwards. Then if we can divide the implied volatilities of call options and put options into two curves and make analysis of the relationship of these two curves, it may be a good indicator for the prediction of HSI. By using the theory of Black-Scholes model to calculate the implied volatility and inspired by the VHSI and the different performance of call and put options, the model of using the implied volatility to predict the trend of HSI is built in the next part.

9.2.3 Model

The model can be divided into two parts. The first part is calculating the implied volatility of options and drawing the curves of implied volatility of call options and put options separately while the second part is to use the information contains in these two curves to predict the trend of HSI.

As to the first part, all the parameters of option from April 2011 to April 2012 which are traded in the Hong Kong market are collected and the time period is 15 minutes. There are total 5398 records for the whole year. Then the equation of Black-Scholes model which has been mentioned above can be expressed:

$$IV = blsiv(P, S, r, T-t, v, y, B) \tag{9.1}$$

The implied volatilities of the nearest five expire dates options which have pricing can be calculated using this equation, because there are almost no records for options whose expire date are too far away.

Then as to the same style options which have the same expire date:

$$IV_B^{T_i} = \sum_{j=1}^{N} \beta_{B_j} IV_{B_j}^{T_i} \Big/ N \qquad (i = 1,2,3,4,5 \qquad B = call \ \ or \ \ put) \tag{9.2}$$

where the parameter, β_{B_j}, is the weight of the implied volatility of jth with style B. Because as to this different style or different strike price, the contribution the implied volatility can make is usually different. So it is necessary to give them a weight.

By using the equations above, to all of these 5398 records, the record of 2012/04/03 16:14 and the record of 2012/04/03 10:29 are chosen as an example. For each record, we can draw the figures like the following:

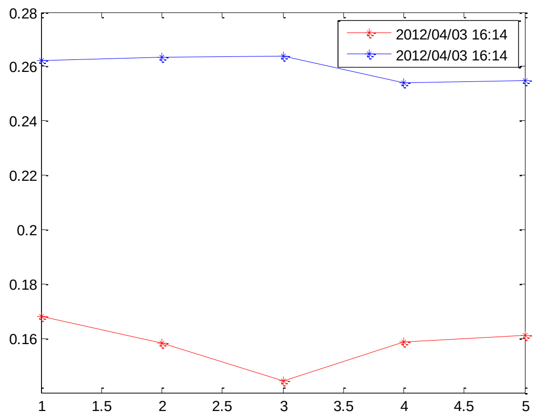

Fig. 9-1 The implied volatility of 2012/04/03 16:14.

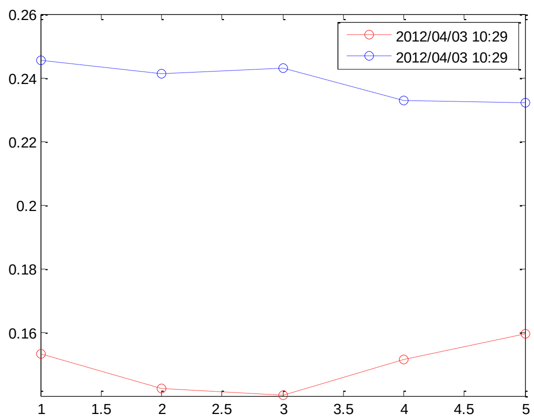

Fig. 9-2 the implied volatility of 2012/04/03 10:29.

Then by adding to these two figures into one picture, we can find that there are changes in the implied volatility of both call and put options.

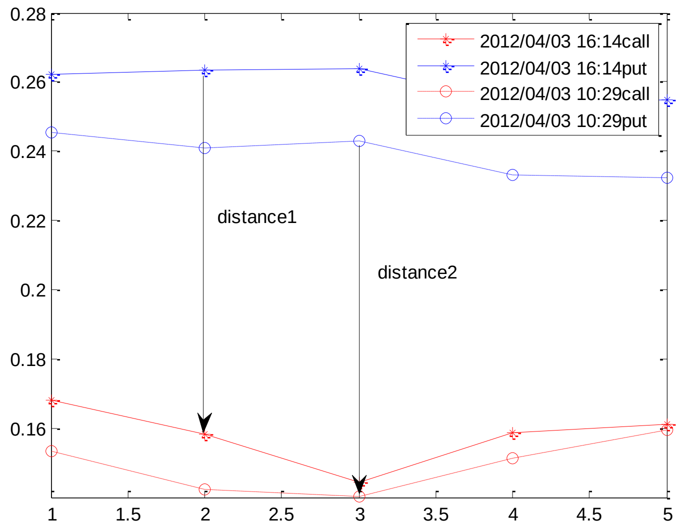

Fig. 9-3 The implied volatility of 2012/04/03 16:14 and 2012/04/03 10:29.

Just as what we have mentioned above, as to the different time, the curves of implied volatilities of call options and put options changes with time. As to the single picture, it can represent the state of the market at one point such as the Fig. 9-1 and Fig. 9-2. As the common sense, we know that if there is a high value of VHSI, it means investors are panic and they worry about the fall of the marker. It is the same with the model in this chapter. If we put the two consequent figures together, it is better for us to analyze the changes of panic situation. The distance such as the distance 1 and distance 2 which have been marked on the figure 3 is called call-put spread in this chapter. As to now, we have completed the first part of the model which is calculating the two curves of implied volatility of call options and put options. In the following, we will complete the second part which has been mentioned in the starting of this part that is using the call-put spread builds model to predict the trend of HSI.

Based on the Equation (9.1) and Equation (9.2), we can get the Fig. 9-1, Fig. 9-2, and Fig. 9-3. The space between the curve of put options and the curve of call options are called spread which can be abstracted as the distance which is marked on the Fig. 9-3. Then using the distance 1 and distance 2 in Fig. 9-3 as examples, they can be written as:

$$\begin{cases} distance1 = D_{5398}^{T_2} = IV_{C_{5398}}^{T_2} - IV_{P_{5398}}^{T_2} \\ distance2 = D_{5378}^{T_3} = IV_{C_{5378}}^{T_3} - IV_{P_{5378}}^{T_3} \end{cases} \tag{9.3}$$

In reality, there is complex relationship between the call-put spread and HSI. It is difficult to use an equation to quantify this kind of relationship. So the model in this chapter instead of making quantitative prediction to the HSI only qualitative prediction is made by using the call put spread.

The mean level, IVC_t , of the implied volatility of call option at time t in figure 4 can be written as:

$$IVC_t = \sum_{i=1}^{5} IV_{C_t}^{T_i} \Big/ 5 \tag{9.4}$$

IVC_t is the level which is the mean value of the different expire date options of implied volatility of call options at the time t. And the parameter $IV_{C_t}^{T_i}$ can by calculated by equation (9.2). In the same way, the level of implied volatility of put options at time t can be written as:

$$IVP_t = \sum_{i=1}^{5} IV_{P_t}^{T_i} \Big/ 5 \tag{9.5}$$

This equation (9.5) has the same meanings with the equation (9.4). By using equation (9.4) and equation (9.5) Fig. 9-4 can be drawn. The parameter in Fig. 9-5 can be written as:

$$DIF_t = IVC_t - IVP_t \tag{9.6}$$

Based on the theory of implied volatility and the characteristics of implied volatility of call options and put options, the model can be abstracted as the judgment of the changing of DIF_t and it can be written as:

$$\begin{cases} DIF_{t+1} - DIF_t \geq 0 \Rightarrow HSI_{t+2} - HSI_{t+1} \geq 0 \\ DIF_{t+1} - DIF_t \leq 0 \Rightarrow HSI_{t+2} - HSI_{t+1} \leq 0 \end{cases} \quad t \geq 0 \tag{9.7}$$

Then this model can use the past two time periods to predict the trend of HSI in the following time period.

The model illustrated above is the basic model which is created in this chapter. Because most scholars think that the near term option can reflect the near term trend of HSI better, the predicting model which use the nearest expire date options are done in this chapter as the benchmark. Then the equation (9.7) can be rewrite as:

$$\begin{cases} DIF_{t+1}^{T_1} - DIF_t^{T_1} \geq 0 \Rightarrow HSI_{t+2} - HSI_{t+1} \geq 0 \\ DIF_{t+1}^{T_1} - DIF_t^{T_1} \leq 0 \Rightarrow HSI_{t+2} - HSI_{t+1} \leq 0 \end{cases} \quad t \geq 0 \tag{9.8}$$

where the parameter $DIF_{t+1}^{T_1}$ is not the mean value of five expire date options. It is only the implied volatility of the nearest expire date option and the equation is written as:

$$DIF_{t+1}^{T_1} = IV_{C_t}^{T_1} - IV_{P_t}^{T_1} \tag{9.9}$$

In fact, no matter the near expire date and the far expire date option, both the implied volatility of them contain the information of the trend of the index. And both of them can be used as the indicator of HSI in a certain degree, though they may have different weight. So the prediction model with weight should have a better performance in theory. Then compared with the mean weight model the equation (9.4) and equation (9.5) can be rewritten as:

$$IVC_t = \frac{\sum_{i=1}^{5} \beta_i \cdot IV_{C_t}^{T_i}}{\sum_{i=1}^{5} \beta_i} \quad and \quad IVP_t = \frac{\sum_{i=1}^{5} \beta_i \cdot IV_{P_t}^{T_i}}{\sum_{i=1}^{5} \beta_i} \tag{9.10}$$

The parameter β_i is the weight to different volatility of different expire dates options. As to this model it is necessary to find the optimal value of β_i. In theory, the β_i should be bigger or equal to the β_{i+1}.

In the following part, we will give the analysis about the rationality of the model and analyze the results of these three models above. The advantages and disadvantages are also will be analyzed in the following and as to these advantages and disadvantages, we will give some advice of future research.

9.3 Results and Analysis

In Fig. 9-4, it is clear to learn about the relationship between the HSI and the mean value of volatilities of call options and put options and in Fig. 9-5, it is the relationship between the difference of volatilities of call options and put options and the HSI. And the time period in both of the two figures are 15 minutes.

From Fig. 9-4, it is obvious that before the point of 3000, there is a downward trend in HSI and during this time period, the parameter IVP is bigger than the parameter IVC. And between the point of 3000 and 6000, there is an upward trend in HSI. As to this time period, in most cases the level of volatility of call options is a little higher than that of put options. What's more, when there is an intense turbulence in the market, the spread between volatility of call options and put options is bigger than usually. This characteristic will be helpful for the future work which may focus on the quantitative prediction.

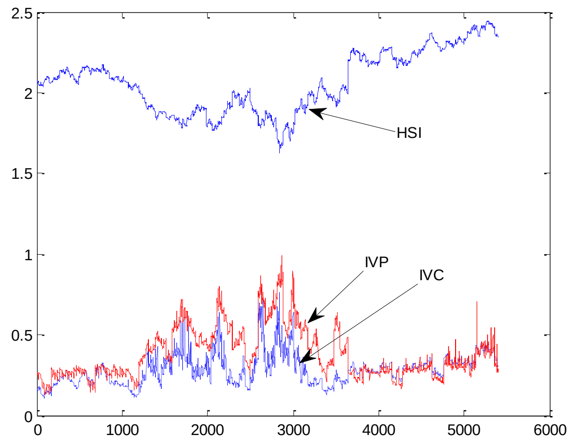

Fig. 9-4 The HSI, IVP and IVC.

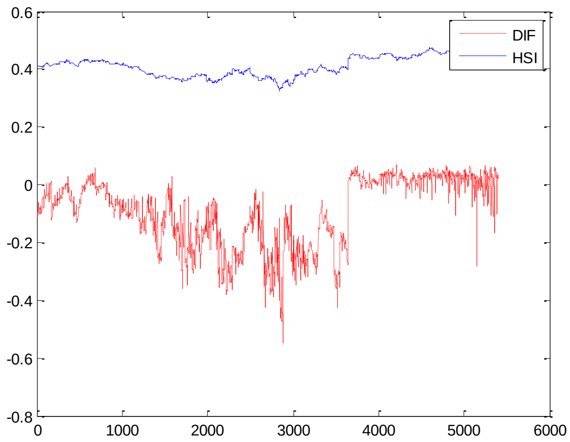

Fig. 9-5 The HSI and DIF.

As to the Fig. 9-5, the above line is the HSI corresponding to the above line in Fig. 9-4 and it is the value of HSI. To compare with the trend of volatilities well, the value of HSI is divided by five thousands. The conclusion can be got from Fig. 9-5 that when there is a downward trend, the value of parameter DIF_t is lower and vice versa.

Base on the analysis to these two figures, it is clear that the conclusion which is assumed above is rational. And the results in the following Table. 9-1 also prove the efficiency of the model in this chapter to predicting the trend of HSI.

Table. 9-1 The results of the prediction.

	Mean	Nearest	Weighted
Hit Rate	54.30%	53.48%	54.34%
Hit Num	2930	2894	2933

The table above shows the result of the three models in this chapter. There is no big difference between these three models. But we still can get the information that the weighted model can get the best results while result of the model only using the nearest term option is the worst which is accordant to the hypothesis we made in the part of model. The value of the parameters in weighted model are $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 1$which is bigger than $\beta_5 = 0.9$. As to this model, the range of these parameters is from 0 to 1 and the step to search to best solution is 0.1. Because the step is a little long, the best solution may be skipped during the process of searching. So this must not be the best solution for the weighted model. If the step length can be reduced into 0.01, it will be more accurate for this model and the better result can be got.

The results in the table and the two figures, Fig. 9-4 and Fig. 9-5, can prove the efficiency of the model well and by analyzing the results. The conclusions are accordant with the hypothesis we made in the former part of the article that the implied volatility of call options and put options is a good indicator for the trend of HSI and what's more, the volatility of options with different expire date have different weights to the function of predicting. However, all of these three models have no big difference while the hit rate and hit number are almost the same. The numbers they can rightly predict are all around 2900 of all 5398 records from April 2011 to April 2012.

There are both advantages and disadvantages in these models. Here the advantages and disadvantages of the weighted model which is the most complete model in these three are analyzed. The model is very simple and it is easy to calculate and to get the value of the parameters. What's more, in predicting the trend of HSI, it can always get a stable result which is around 54%. It is a significant benchmark for the research future and it is a good result for the trading in reality. Though it is a simple and useful model, it is not enough to mine all the information involved in the implied volatility of options. The better result can be got if the model is better. One shortcoming for this model is that the implied volatility for any time is the mean value of all the options which have trading records. This is not accurate, because of the existing of the volatility smile and a lot of work has been done in this field. The volatility of the deeply out-of-the-money is always higher and the information in deeply out-of-the-money may be not accurate, so the method of calculating the mean value will leading to an error while the volatility of deeply out-of-the-money options have the same weighted with the other options. The method to solve this problem is not difficult. Just take the volatility smile into accounting and as to the volatilities of deeply out-of-the-money options, give them slight weights to reduce the function they can make. So for the future research, if the volatility smile can be taken into accounting, the results will be better.

The other shortcoming of the models in this chapter is that the prediction is qualitative which means that only the direction of the HSI can be predicted. Though it already a good result for the trading in reality, the quantitative prediction is perfect for the trading. But compared with the qualitative prediction, the quantitative prediction is much more difficult, because the relationship between the delta of HSI and the delta of volatility is very complex, even it may be not possible to be express in equations. But it is still a good research direction for future research.

What's more, because of the limitation of the data in this research, the time period is 15 minutes and the authors did not try any other time periods. It may be not a good time period for prediction of Hong Kong market. In future, the other time period can be tested. It may lead to a good result too. In a word, though the research in This chapter is very significant and important, there still a lot of shortcomings and a lot of work still can be done in the future to make the model complete and to make the prediction accuracy.

9.4 Conclusions

Inspired by the intensive negative relationship between the VHSI and HSI and the different performances of the implied volatilities of put options and call options, This chapter is the first time to divide the VHSI into the volatilities of call options and volatility of put options and use the changes of the position of these two volatilities which means the call-put spread in This chapter as the indicator to predict the trend of HSI. The call-put spread is complex and in this chapter it is abstracted into the distance between the volatilities of call options and put options. It can make the model simple.

Three models, the nearest term volatility, the mean value and the weighted volatilities models are built in this chapter to test the efficiency of call-put spread in predicting the trend of HSI. The method of using the call-put spread to predict the index of market is first stated in this chapter and the results show that the models are rational. It opens the direction to calculate the volatilities of call options and put options separately which is different from the VHSI that combines them together and it is also the first time to dig the information contained in the position of the volatilities of call and put options. All of these three models have similar results and the results in the last part shows the correctness of these models and the result proves that there is a lot of information in the call-put spread and it is possible to make future research.

Moreover, both the advantages and disadvantages are analyzed in the former part and based on the analysis the author gives the direction of future research. Though only the qualitative prediction is made in this chapter, it opens a new direction for the predicting of index and the possibility of making quantitative prediction is also given in the former part. And it is a new topic for the research of volatility of options. The research in this chapter not only makes the significant contribution to the research area but also to the trading in reality.

References

Ahn Dong-Hyun, Jacob Boundoukh, Matthew Richardson and Robert F. Whitelaw (1997). Optimal Risk Management Using Options. *Working paper of National Bureau of Economic Research*.

Andersen G.Torben, Luca Benzoni and Jesper Lund (2002). An Empirical Investigation of Continuous-Time Equity Return Models. *Journal of Finance* **57**(3): 1239-1284.

Andersen Torben G., Tim Bollerslev and Francis X. Diebold (2003). Handbook of Financial Econometrics, chap. Parametric and Nonparametric Volatility Measurement.

Arditti Fred D. and John Kose (1980). Spanning the State Space with Options. *Journal of Financial Quantitative Analysis* **15**: 1-9.

Asaia Manabu and Michael McAleer (2009). The Structure of Dynamic Correlations in Multivariate Stochastic Volatility Models. *Journal of Econometrics* **150**(2): 182-192.

Bates DS (2000). Post-87 Crash Fears in S&P 500 Futures Options. *Journal of Econometrics* **94**: 181-238.

Beckers S. (1981). Standard deviations implied in option prices as predictors of future stock price variability. *Journal of Banking and Finance* **5**(3): 363–381.

Berestycki Henri, Jerome Busca and Lgor Florent (2004). Computing the Implied Volatility in Stochastic Volatility Models. *Communications on Pure and Applied Mathematics* **LVII**: 1352–1373.

Bessembinder Hendrik, Jay Coughenour, Paul Seguin and Margaret Monroe Smoller (1995). Mean Reversion in Equilibrium Asset Prices: Evidence from the Futures Term Structure. *Journal of Finance* **50**(1): 361-375.

Black Fischer (1990). Mean Reversion and Consumption Smoothing. *Review of Financial Studies* **3**(1): 107-114.

Black Fischer and Myron Scholes (1973). The Pricing of Options and Corporate Liabilities. *The Journal of Political Economy* **81**: 637-654.

Bollerslev Tim, Michael Gibson and Hao Zhou (2007). Dynamic Estimation of Volatility Risk Premia and Investor Risk Aversion form Option-Implied and Realized Volatility. *Working Paper of National Bureau of Economic Research*.

Bollerslev Tim and Hao Zhou (2002). Estimating Stochastic Volatility Diffusion Using Conditional Moments of Integrated Volatility. *Journal of Econometrics* **109**: 33-65.

Bollerslev Tim and Hao Zhou (2003). Volatility Puzzles: A Unified framework for Gauging Return-Volatility Regressions. *Working Paper of National Bureau of Economic Research*.

Bookstaber Richard and Roger Clarke (1984). Option Portfolio Strategies: Measurement and Evaluation. *Journal of Business* **57**(4): 469-492.

Bookstaber RichardRoger and Roger Clarke (1983). An Algorithm to Calculate the Return Distribution of Portfolios with Option Positions. *Management Science* **29**(4): 419-429.

Breeden D. and R. Litzenberger (1978). Price of state-contingent claims implicit in option price. *Journal of Business* **52**: 621-651.

Britten-Jones Mark and Anthony Neuberger (2000). Option Prices, Implied Price Processes, and Stochastic Volatility. *Journal of Finance* **55**: 839-866.

Carr Peter and Dilip Madan (1998). Towards a theory of volatility trading, London.

Carr Peter and Liuren Wu (2009). Variance Risk Premiums. *Review of Financial Studies* **22**: 1311-1341.

Cecchetti Stephen, Pok-Sang Lam and Mark Nelson (1990). Mean Reversion in Equilibrium Asset Prices. *The American Economic Review* **80**(3): 398-418.

Chernov Mikhail and Eric Ghysels (2000). A Study Towards a Unified Approach to the Joint Estimation of Objective and Risk Neutral Measures for the Purpose of Option Valuation. *Journal of Financial Economics* **56**: 407-458.

Christoffersen Peter, Steve Heston and Kris Jacobs (2006). The Shape and Term Structure of the Index Option Smirk: Why Multifactor Stochastic Volatility Models Work so Well. *Management Science* **55**: 1914-1932.

Coleman Thomas, Yuying Li and Arun Verma (1999). Reconstructing the Unknown Local Volatility Function. *Journal of Computational Finance* **2**(3): 77-100.

Cox J (1975). Notes on Option Pricing I: Constant Elasticity of Diffusions. *Stanford University*, California.

Crépey Stéphane (2003). Calibration of the Local Volatility in a Generalized Black-Scholes Model using TIKHONOV Regularization. *SIAM Journal of Mathematical Analysis* **34**(5): 1183-1206.

Demeterfi Kresimir, Emanuel Derman, Michael Kamal and Joseph Zou (1999). A Guide to Volatility and Variance Swaps. *Journal of Derivatives* **6**(4): 9-32.

Dumas Bernard, Jeff Fleming and Robert E. Whaley (1998). Implied Volatility Functions: Empirical Tests. *Journal of Finance* **LIII**(6).

Dupire Bruno (1994). Pricing with a smile. *Risk* **7**: 18-20.

Emanuel David and James MacBeth (1982). Further Results of the Constant Elasticity of Variance Call Option Pricing Model. *Journal of Financial and Quantitative Analysis* **4**: 533-553.

Emanuel Derman and Iraj Kani (1994a). The Volatility Smile and Its Implied Tree. *Quantitative Strategies Research Notes*.

Emanuel Derman and Iraj Kani (1994b). Riding on the smile. *Risk* **7**: 32–39.

Engle Robert and Andrew Patton (2001). What Good Is a Volatility Model? *Quantitative Finance* **1**: 237-245.

Eric Benhamou, Gobet Emmanuel and Miri Mohammed (2010). Time dependent Heston model. *SIAM Journal on Financial Mathematics* **1**: 289-325.

Ernst and Young (1999). Internet Shopping: An Ernst and Young Special Report. *Ernst and Young LLP*.

Feng Jin, Forde Martin and Jean-Pierre Fouque (2010). Short-Maturity Asymptotics for a Fast Mean-Reverting Heston Stochastic Volatility Model. *SIAM Journal on Financial Mathematics* **1**(1): 126-141.

Fouque Jean-Pierre, George Papanicolaou and K. Ronnie Sircar (2000). Mean-Reverting Stochastic Volatility. *International Journal of Theoretical and Applied Finance* **3**(1): 101.

Heston Steven (1993). A Closed Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options. *Review of Financial Studies* **6**(2): 327-343.

Hobson David (2004). Stochastic Volatility Models, Correlation, and the q-Optimal Measure.

Mathematical Finance **14**(4): 537-556.

Ito Kiyoshi (1951). On stochastic differential equations. *American Mathematical Society* **4**: 151.

Jose Da Fonsecaab, Grasselliac Martino and Tebaldid Claudio (2008). A Multifactor Volatility Heston Model. *Quantitative Finance* **8**(6): 591-604.

Josep Perelló, Masoliver Jaume and Anento Napoleón (2004). A Comparison between Several Correlated Stochastic Volatility Models. *Physica A: Statistical Mechanics and its Applications* **344**(1): 134-137.

Joshua D. Coval and Shumway Tyler (2001). Expected Option Return. *The Journal of Finance* **LVI**(3).

Kahl Christian and Peter Jackel (2006). Fast Strong Approximation Monte Carlo Schemes for Stochastic Volatility Models. *Quantitative Finance* **6**(6): 513-536.

Keynes John Maynard (1936). The General Theory of Employment, Interest and Money. *Macmillan Cambridge University Press*, New York.

Lee Roger W. (2004). Implied Volatility: Statics, Dynamics, and Probabilistic Interpretation. *In Recent Advances in Applied Probability*.

Lin Guanzhi (2010). Having a Date With Option. *Jucaizixun*, Taiwan.

Luo Xingguo and Jin Zhang (2010). The Term Structure of VIX. 2010 Financial Management Association (FMA) European Conference. Hamburg, Germany.

Mark Rubinstein (1994). Implied Binomial Trees. *Journal of Finance* **49**: 771–818.

Merton Robert C., Myron S. Scholes and Mathew L. Gladstein (1978). The Return and Risk of Alternative Call Option Portfolio Investment Strategies *The Journal of business* **51**(2): 183-242

Merton Robert C., Myron S. Scholes and Mathew L. Gladstein (1982). The Returns and Risks of Alternative Put-Option Portfolio Investment Strategies. *The Journal of Business* **55**(1): 1-55.

Moodley Nimalin (2005). The Heston Model: a Practical Approach with Matlab Code. *University of the Witwatersrand*, Johannesburg.

Ncube Moeti (2009). Stochastic Models and Inferences for Commodity Futures Pricing. Tallahassee, Florida State University.

Ross Sheldon M. (2007). Introduction to Probability Models.

Ross Stephen A. (1976). Option and efficiency. *Quarterly Journal of Economics* **90**(1): 75-89.

Saikat Nandi (1998). How Important Is the Correlation between Returns and Volatility in A stochastic Volatility Model? Empirical Evidence from Pricing and Hedging in the S&P 500 Index Options Market. *Journal of Banking and Finance* **22**: 589-610.

Sepp Artur (2002). Pricing Barrier Options under Local Volatility.

Sepp Artur (2008). Pricing Options on Realized Variance in the Heston Model with Jumps in Returns and Volatility. *Journal of Computational Finance* **11**(4): 33-70.

Shehata Emad Abd Elmessih and Sahra Khaleel A. Mickaiel (2012). IMVOL: Stata module to compute Implied Volatility in Black-Scholes European Option Pricing Model. *Economic Research Division of the Federal Reserve Bank of St. Louis*.

Sylvia Fruhwirth-Schnatter and Sogner Leopold (2008). Bayesian Estimation of the Multi-Factor Heston Stochastic Volatility Model. *Communications in Dependability and Quality Management* **11**(4): 5-25.

Taylor Stephen, Pradeep Yadav and Yuanyuan Zhang (2010). The Information Content of Implied Volatilities and Model-free Volatility Expectations: Evidence from Options Written on Individual Stocks. *Journal of Banking and Finance* **34**: 871-881.

Zhang Jin and Yi Xiang (2008). The Implied Volatility Smirk. *Quantitative Finance* **8**: 263-284.

Subject Index

O

P

R

S

T

V

W

Biographies of Four Authors of the Book

Zhou Shifei received his master degree in Computer Science and Technology from Central South University in Changsha, China. He is currently pursuing a PhD degree at Department of Management Sciences of City University of Hong Kong. His research interest mainly focuses on financial behavior, financial derivatives, and financial engineering.

Hao Wang received his bachelor degree in Computer Science and Technology from Northwestern Poly-technical University in Xi'an, China. He is currently pursuing a PhD degree at Department of Management Sciences of City University of Hong Kong. His research interest mainly focuses on risk management, financial derivatives, and financial engineering.

Kin Keung Lai received his PhD degree at Michigan State University in USA in 1977 and is currently a Chair Professor of Management Science at City University of Hong Kong. Prior to his current post, he was a Senior Operational Research Analyst at Cathay Pacific Airways and an Area Manager for Marketing Information System at Union Carbide Eastern. He is the president of the Asia-Pacific Industrial Engineering and Management Society, the general secretary of Hong Kong Operational Research Society and a council member of the International Federation of Operations Research Societies. He is also a co-editor of 8 journals including Journal of the Operational Research Society and the managing editor of book series "Lecture Notes in Decision Science". He has published five monographs and over 100 journal papers. His main research interests include supply chain and operation management, forecasting, computational intelligence and risk analysis.

Jerome Yen received his PhD degree at University of Arizona with major Systems Engineering and Management Information Systems. He is now a Professor of Accounting and Finance department of Tung Wah College. Prior to his current post, he was a senior member of Academic Advisory Committee PRMIA, the president of Asia Pacific Association of Financial Engineering, and Co-Chair of First Asia Pacific Conference on Financial Engineering held in Hong Kong 2008. His teaching areas include Derivatives, Investment Analysis and Portfolio Management, Risk Management, Exotic Options and Structured products, and China Financial Markets. He has published over 50 journal papers. His main research interests include financial engineering, investment management, market and credit risk management, financial product development, trading strategies and hedge funds.